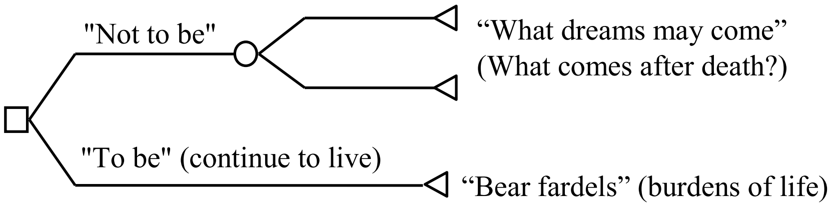
**IE5203 Decision Analysis**
**Solutions to Chapter 4 Exercises**

**P4.1** Hamlet's Decision Model:

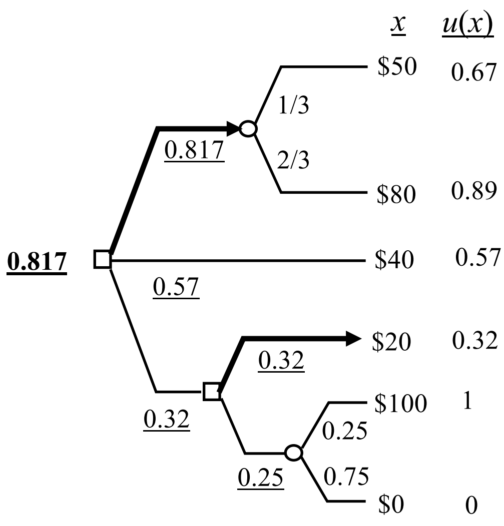



**P4.2**

- If we let $u(\$100) = 1$, and $u(\$0) = 0$, then the preference probabilities are the same as the utilities.

| Value ($\$x$) | $u(x)$ |
|---|---|
| 0 | 0.00 |
| 10 | 0.17 |
| 20 | 0.32 |
| 40 | 0.57 |
| 50 | 0.67 |
| 80 | 0.89 |
| 90 | 0.95 |
| 100 | 1.00 |

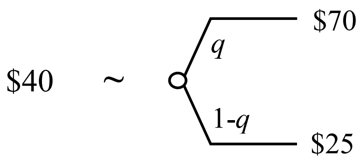
- Rolling back the decision tree and computing the expected utilities:




- The optimal decision is to take the first alternative which has the maximum expected utility of 0.817.
- The certainty equivalent is obtained by converting the expected utility back to its equivalent dollar value.
- Hence Kim's CE for the opportunity = $u^{-1}(0.817) \approx \$70$ by interpolation on the table.

**P4.3**

- If we let $u(\$100) = 1$, and $u(\$0) = 0$, then the preference probabilities are the same as the utilities.
- We want to find the value of probability $q$ such that Connie's personal indifferent selling price or certainty equivalent for the deal is equal to $40.




$$u(\$40) = q\,u(\$70) + (1 - q)\,u(\$25)$$
$$0.5 = 0.8\,q + 0.3\,(1 - q)$$
$$q = 0.4$$

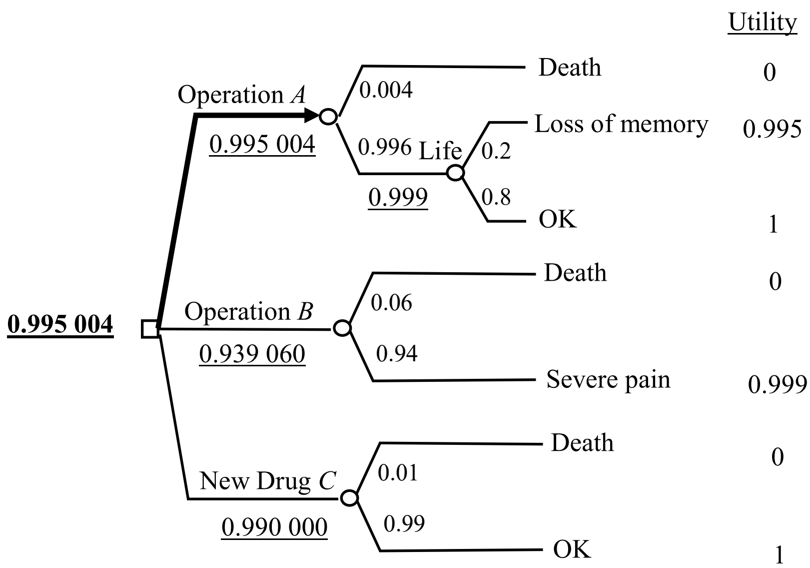
**P4.4**

**(*a*)**

- Note that the best outcome is "OK", and the worst outcome is "Death".
- Let $u(\text{OK}) = 1$ and $u(\text{Death}) = 0$.
- Based on Dr. Tan's preferences, the utility for the various outcomes are as follows:

| Outcome | Utility |
|---|---|
| OK | 1 |
| Severe pain (SP) | 0.999 |
| Loss of memory (LOM) | 0.995 |
| Death (D) | 0 |

- Using Dr. Tan's utilities in the decision tree:



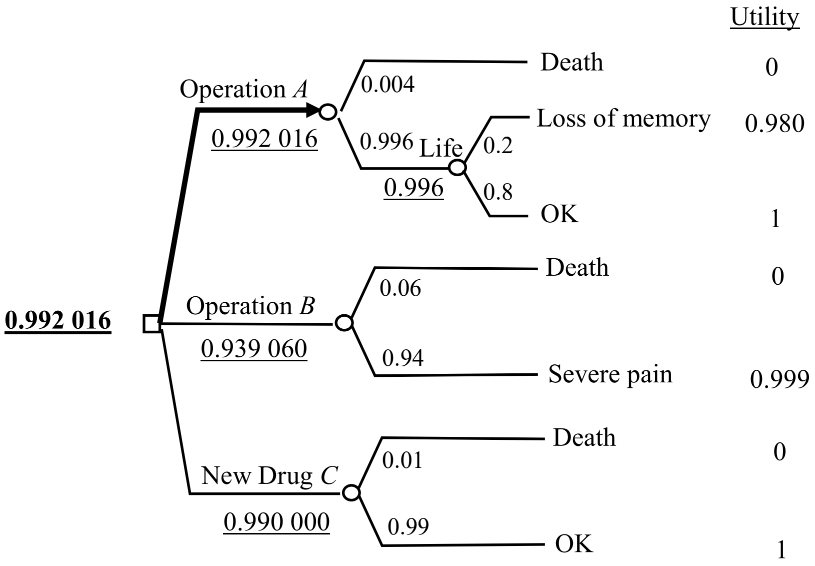

- By rolling back the decision tree using Dr. Tan's preferences, the best alternative is Operation *A* with a maximum expected utility of 0.995004

(*b*)

- Based on Mr. Goh's preferences, and again assuming that $u$(OK)=1, and $u$(Death) = 0, the utility for the various outcomes are as follows:

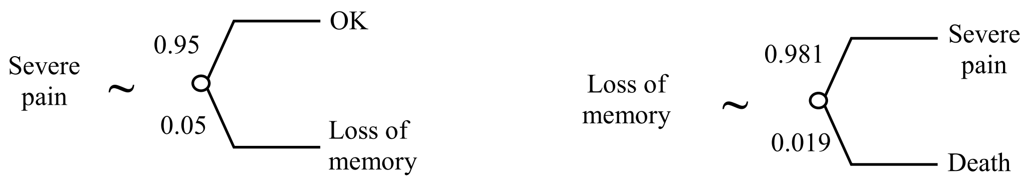
| Outcome | Utility |
|---|---|
| OK | 1 |
| Severe pain (SP) | 0.999 |
| Loss of memory (LOM) | 0.980 |
| Death (D) | 0 |

- Using Mr. Goh's utilities in the decision tree:




- Based on Mr. Goh's preferences, the best alternative is Operation A with a maximum expected utility of 0.992 016.

(*c*) Given




$u$(SP) = 0.95 $u$(OK) + 0.05 $u$(LOM) = 0.95 + 0.05 $u$(LOM)

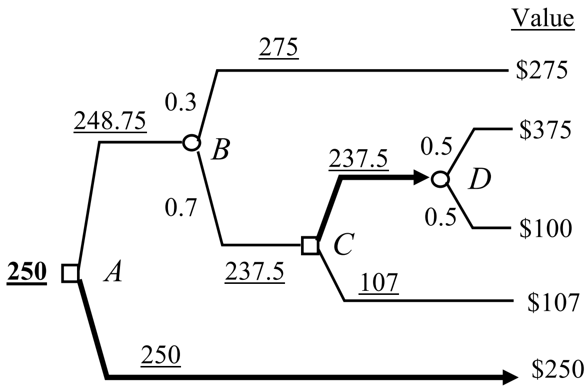
$u$(LOM) = 0.981 $u$(SP) + 0.019 $u$(D) = 0.981$u$(SP)

$\Rightarrow$ $u$(SP) = 0.999 and $u$(LOM) = 0.980

Therefore, the optimal decision remains unchanged for Mr. Goh.

**P4.5**

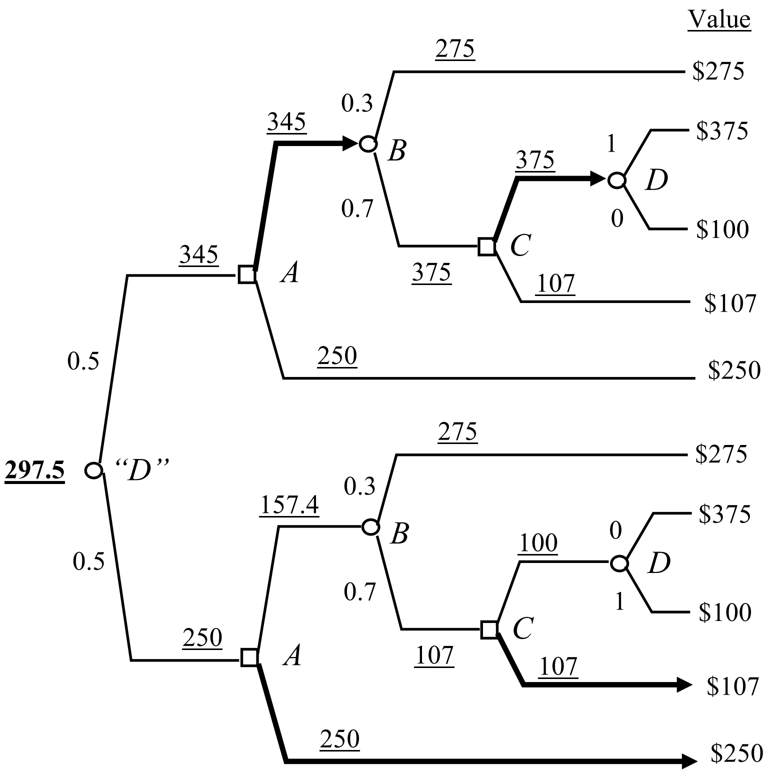
(***a***) Jeanne's decision tree:




- Jeanne is risk neutral. This means that she makes her decision based on maximum expected dollar values.
- Also, Risk Neutral ⇒ Certainty Equivalent = Expected dollar value.
- Jeanne's certainty equivalent for decision *A* = **$250**.

(***b***) Value of Clairvoyance on uncertainty *D* before Decision *A*.

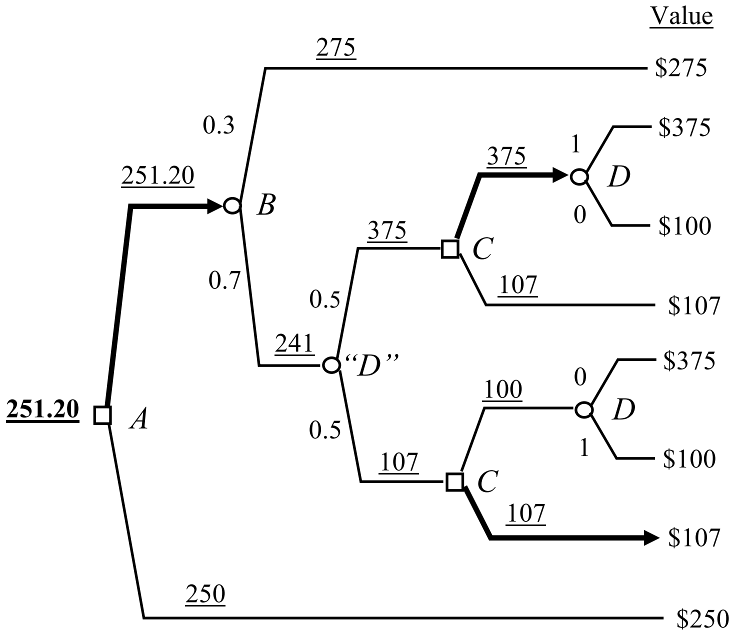
- If Jeanne receives free perfect information on the outcome of *D* just before she makes decision *A*, then the decision model with free clairvoyance on *D* before decision *A* is:




- Note that we have redrawn the decision tree so that chance node "*D*" appears before decision *A*.
- Certainty equivalent when there is free clairvoyance on event *D* before decision *A* = $ 297.50
- Certainty equivalent when there is no information (from part (*a*)) = $ 250.
- Value of Clairvoyance on event *D* before decision *A* = $ 297.50 – $250 = $ 47.50
- Hence the most Jeanne should pay for clairvoyance on event *D* before decision *A* = **$ 47.50**

**(*c*)** Value of Clairvoyance on uncertainty *D* before decision *C* but after decision *A*.

- If Jeanne receives free perfect information on the outcome of uncertain event *D* after making decision *A*, but before making decision *C*, then the decision model is:




- Note that we have redrawn the decision tree so that chance node "*D*" appears after decision *A* but before decision *C*.

- Also *D* and *B* are independent events. Hence we can also draw information node "*D*" before node *B* and obtain the same answer, but the decision tree would be larger in size.

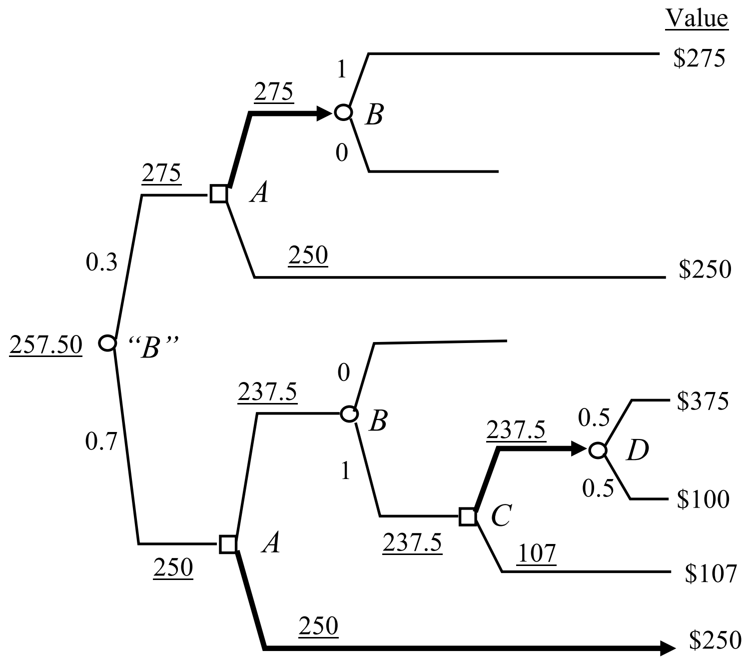
- Certainty equivalent when there is free clairvoyance on event *D* before making decision *C*, but after making decision *A* = $251.20.

- Certainty equivalent when there is no information (from part (*a*)) = $250.

- Value of Clairvoyance on event *D* before making decision *C*, but after making decision *A* = $251.20 – $250.00 = $1.20

- Hence the most Jeanne should pay for clairvoyance on event *D* before making decision *C*, but after making decision *A* = **$1.20** (***d***) Value of Clairvoyance on uncertainty *B* before decision *A*.

- If Jeanne receives free perfect information on the outcome of uncertain event *B* before making decision *A*, then the decision model is:




- Certainty equivalent when there is free perfect information on event *B* before decision *A* = $257.50

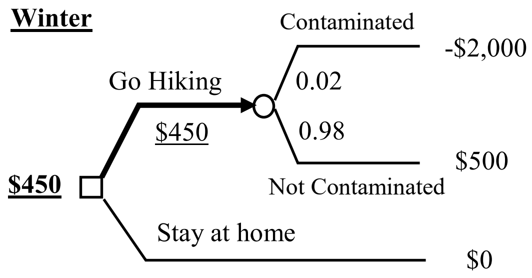
- Certainty equivalent when there is no information (from part (*a*)) = $250.00

- Value of Clairvoyance on event *B* before making decision *A* = $257.50 – 250.00 = $7.50

- Hence the most Jeanne should pay for clairvoyance on event *B* before making decision *A* = **$7.50**

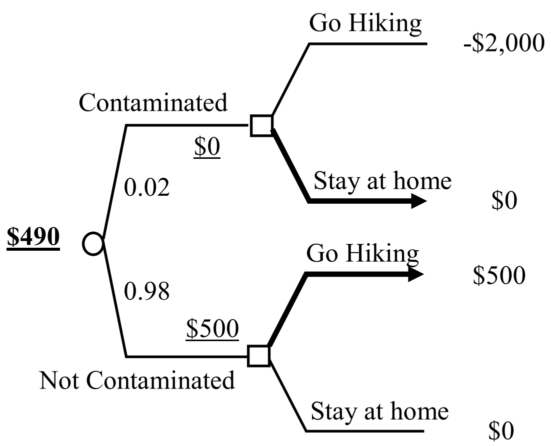
**P4.6** Dorothy is risk neutral.

(*a*) Dorothy's decision tree is as follows:




- Dorothy's best alternative is to go hiking at Yosemite.
- Certainty equivalent for the best alternative = $450.00

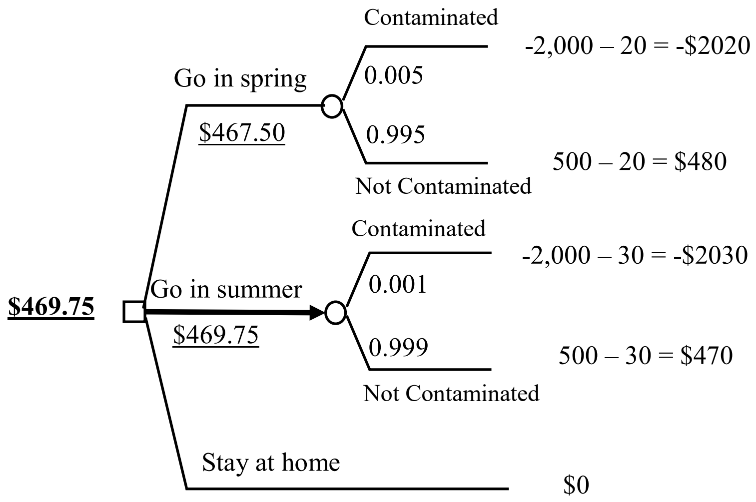
(*b*) Decision model with free clairvoyance on contaminated water:




- Note that we have omitted the redundant event with ZERO-ONE probabilities in the above decision tree and treated the event of water contamination as "happening" before the decision to go hiking or not.

- Certainty Equivalent with free clairvoyance on the presence of contaminated water = $490.00
- Certainty Equivalent with no information = $450.00

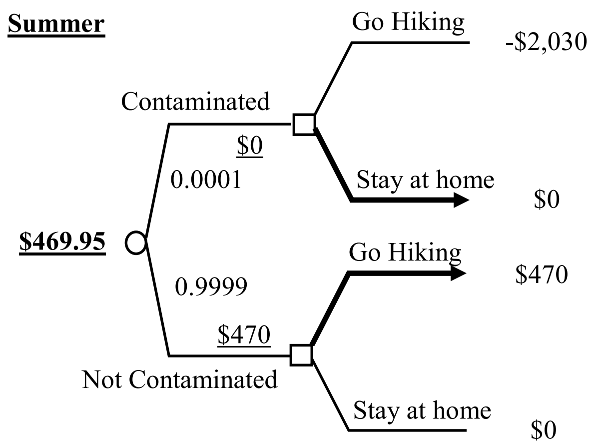
- Dorothy's expected value of perfect information (EVPI) on the presence of contaminated water = $490.00 – 450.00 = **$40.00** (*c*) Decision tree to decide on which season to go:




- Dorothy's best alternative is to go during summer which has a maximum certainty equivalent of $469.75.

(*d*) Dorothy's alternatives are now just "Go hiking (in summer)" or stay at home. Hence only the second and third branches of the above decision tree are applicable.

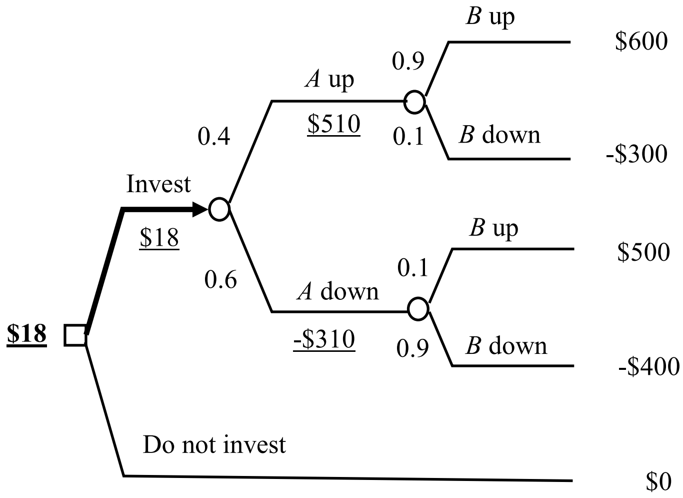
- Decision model with free clairvoyance:

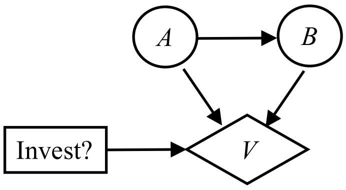



- Certainty equivalent with free clairvoyance = $469.95.
- Certainty equivalent with no clairvoyance for summer alternative = $469.75
- Dorothy's value of clairvoyance on contaminated water = $469.95 - $469.75 = **$0.20**

**P4.7** Al is risk neutral

**(*a*)** Base decision model:

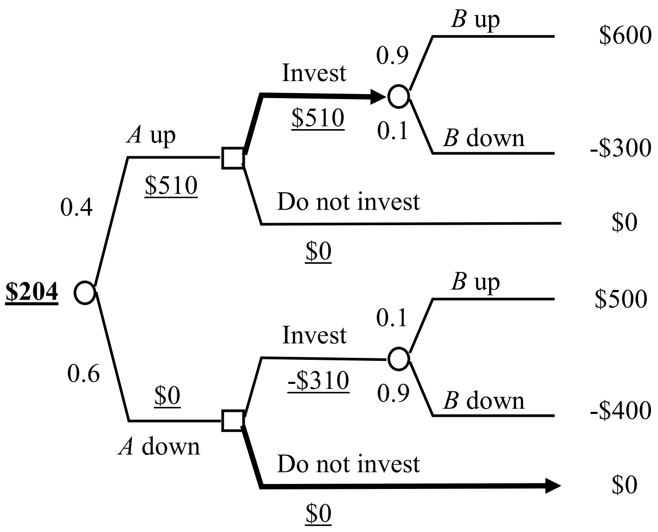


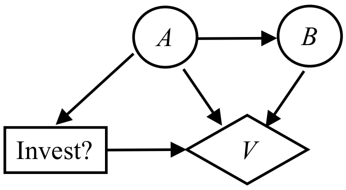




- Al's best decision is to invest.
- Al's certainty equivalent for the deal = $18.00

**(*b*)** Value of Clairvoyance on stock *A*:

- Decision model with free clairvoyance on the performance of stock A:

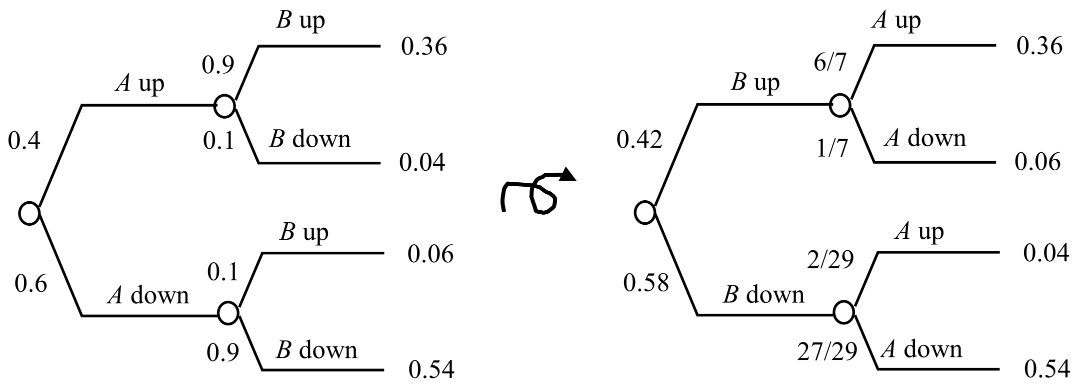


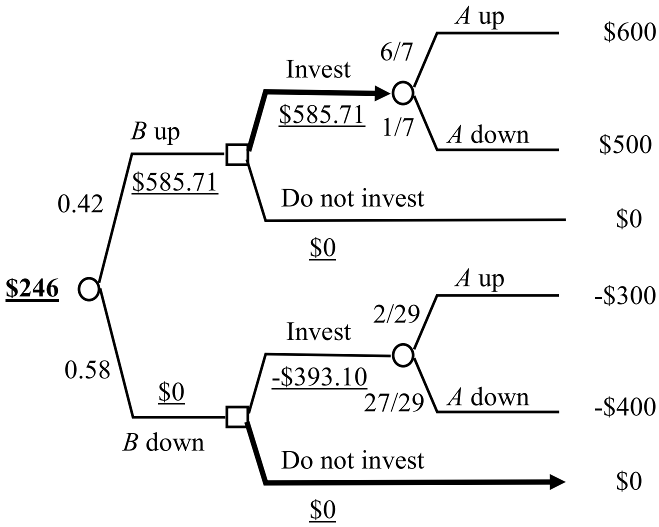

- CE with free clairvoyance on the performance of Stock *A* = $204.00
- CE with no clairvoyance = $18.00
- Hence Al's value of clairvoyance on the performance of Stock *A* = $204.00 - $18.00 = **<u>$186.00</u>**

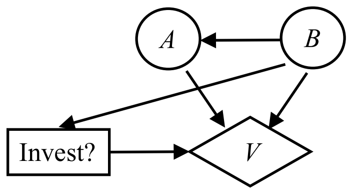
(*c*) Value of Clairvoyance on stock *B*:

- When there is free clairvoyance on *B*, we need to draw the decision tree with node *B* appearing before the investment decision node which is before node *A*. Hence B will appear before *A* in the decision tree.
- As *A* and *B* are not independent, we need to flip the tree between *A* and *B* and find *p*(*A* | *B*) for the decision tree.
- Note that if *A* and *B* were independent, then tree flipping is not necessary. See for example P4.5 of this exercise.




- Decision tree with free clairvoyance on *B*:

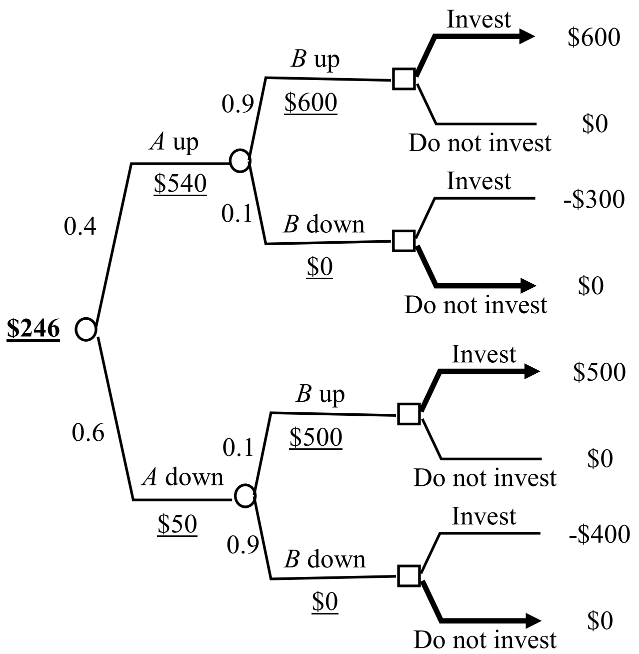


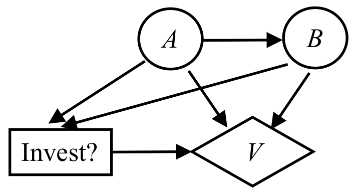




- Certainty equivalent with free clairvoyance on the performance of Stock *B* = $246.00
- Certainty equivalent with no clairvoyance = $18.00
- Hence Al's value of clairvoyance on the performance of Stock *B* = $246.00 - $18.00 = **<u>$228.00</u>**

(***d***) Joint Value of Clairvoyance on Stock *A* and Stock *B*:

- Decision model with free clairvoyance on the performance of both stocks *A* and *B*:



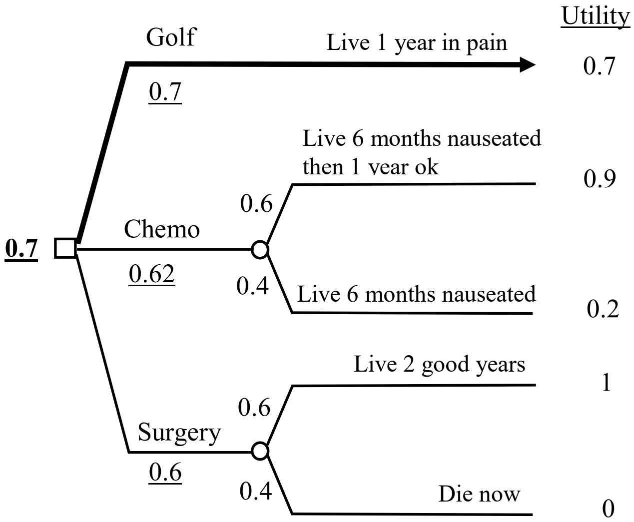




- Certainty Equivalent with free clairvoyance on the performance of both stock *A* and Stock *B* = $246.00
- Certainty Equivalent with no clairvoyance = $18.00
- Al's Joint Value of Clairvoyance on the performance of Stock *A* and Stock *B* = $246.00 – $18.00 = **$228.00**

**P4.8**

- If we let $u$(Live 2 good years) = 1, and $u$(Die now) = 0, then Roy's preference probabilities for the various outcomes are the same as his utilities.

- Roy's decision problem:




**(*a*)** Roy should play golf which has the maximum expected utility of 0.7.

**(*b*)** We cannot calculate the value of information on anything here because we do not have Roy's equivalent dollar values for the outcomes.

**(*c*)** Roy might want the expected value of perfect information so that he would know how much he should spend for any form of information collection activities concerning any of the uncertainties he faces.