P5.1

(*a*)	$B \perp C \mid F$	False	*B-D-C* is blocked but *B-A-C* is not blocked.
(*b*)	$B \perp C \mid A$	True	*B-A-C* is blocked and *B-D-C* is blocked.
(*c*)	$B \perp C \mid \{A, E\}$	False	*B-A-C* is blocked but *B-D-C* is not blocked.
(*d*)	$B \perp C \mid \{A, D\}$	False	*B-A-C* is blocked but *B-D-C* is not blocked.
(*e*)	$A \perp D \mid \{B, G\}$	False	*A-B-D* is blocked but *A-C-D* is not blocked.
(*f*)	$A \perp D \mid \{B, C\}$	True	*A-B-D* is blocked and *A-C-D* is blocked.
(*g*)	$E \perp F \mid \{B, C\}$	True	*F-A-B-D-E* is blocked and *F-A-C-D-E* is blocked.

P5.2

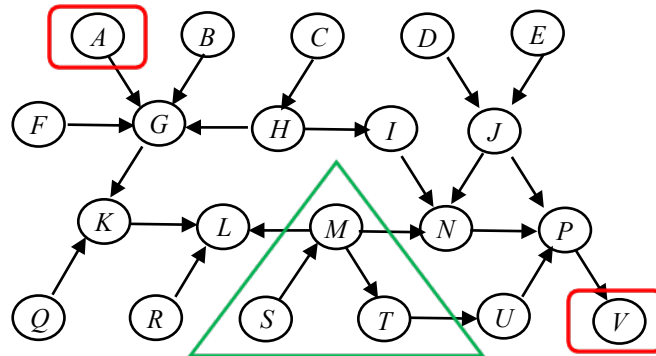**(a)** $\{A\} \perp \{V\} \mid \{H, L, R\}$ **False**
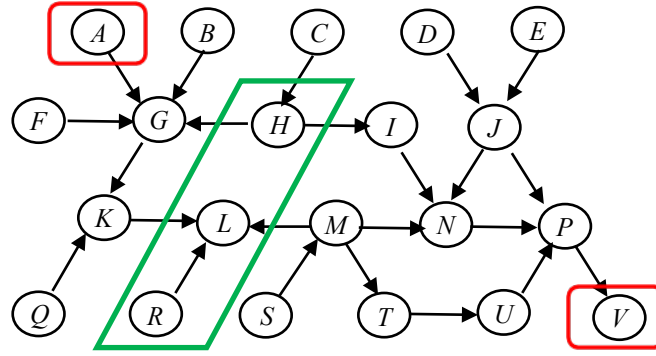
- There exists a path *A-G-K-L-M-N-P-V* that is not blocked as no node along this path satisfies either Condition (1) or (2):
  - *G* is H2T but $G \notin \{H, L, R\}$.
  - *K* is H2T but $K \notin \{H, L, R\}$.
  - *L* is H2H but $L \in \{H, L, R\}$.
  - *M* is T2T but $M \notin \{H, L, R\}$.
  - *N* is H2T but $N \notin \{H, L, R\}$.
  - *P* is H2T but $P \notin \{H, L, R\}$.
- Since there exists a path from $\{A\}$ to $\{V\}$ that is not blocked by $\{H, L, R\}$ it follows that the statement is False.

**(b)** $\{A\} \perp \{V\} \mid \{M, S, T\}$ **True**

- All paths from $\{A\}$ to $\{V\}$ are blocked by $\{M, S, T\}$:

  - *A-G-H-I-N-P-V* is blocked   // *G* is H2H and *G, K* & $L \notin \{M,S,T\}$
  - *A-G-H-I-N-J-P-V* is blocked   // *G* is H2H and *G, K* & $L \notin \{M,S,T\}$
  - *A-G-H-I-N-M-T-U-P-V* is blocked   // *G* is H2H and *G, K* & $L \notin \{M,S,T\}$
  - *A-G-K-L-M-N-P-V* is blocked   // *M* is T2T and $M \in \{M,S,T\}$
  - *A-G-K-L-M-N-J-P-V* is blocked   // *M* is T2T and $M \in \{M,S,T\}$
  - *A-G-K-L-M-T-U-P-V* is blocked   // *M* is T2T and $M \in \{M,S,T\}$

- Since all the paths from $\{A\}$ to $\{V\}$ are blocked by $\{M, S, T\}$ it follows that the statement is True.
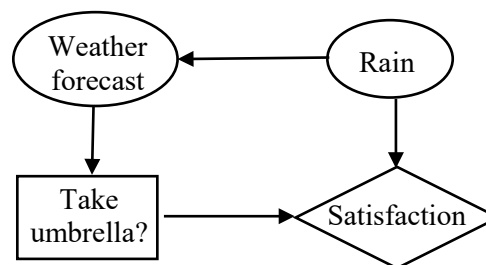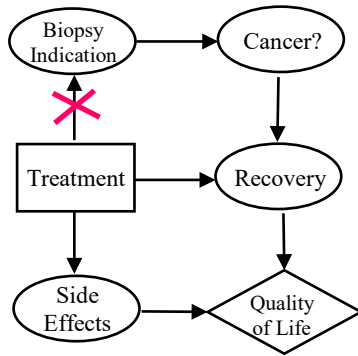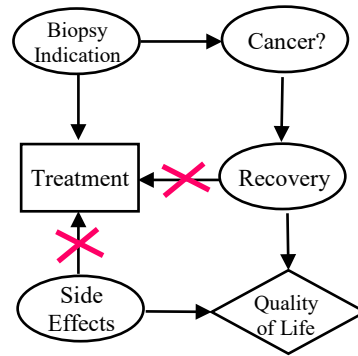
P5.3

(*a*)
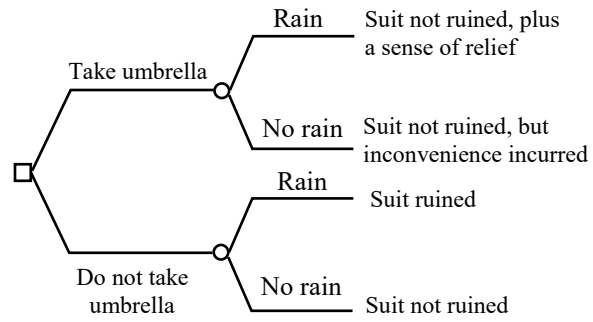
(*b*)
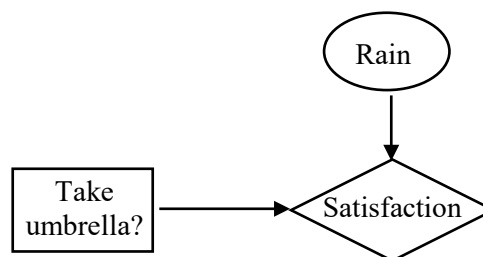
(*c*)

P5.4   The correct influence diagram is (*c*). The other three diagrams are not valid due to the offending arcs indicated.

(***a***)

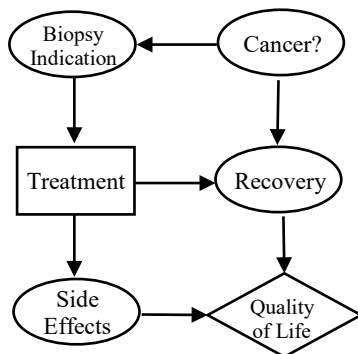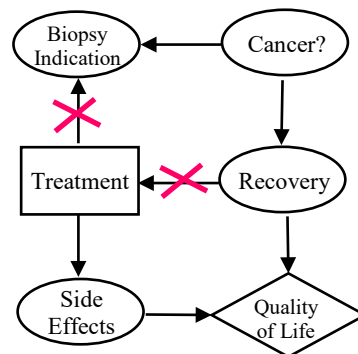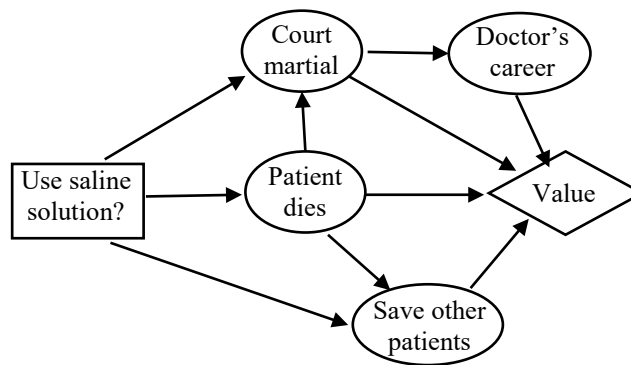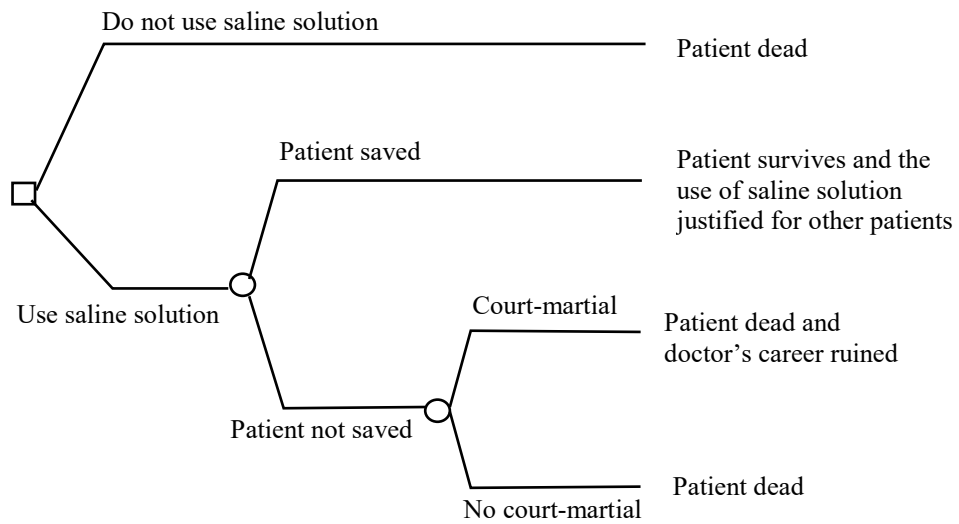(***b***)

(***c***)

(***d***)

P5.5

Do not use saline solution — Patient dead

Patient saved — Patient survives and the use of saline solution justified for other patients

Use saline solution

Court-martial — Patient dead and doctor's career ruined

Patient not saved

No court-martial — Patient dead

Court martial → Doctor's career

Use saline solution? → Patient dies

Patient dies → Value

Save other patients

Value

P5.6

(*a*)

D   A   V

(*b*)

B   A   V

(*c*)

B   C   A   V

(*d*)

B   A   C   V

(*e*)

Test   Test results   Drill   Seismic structure   Amount of oil   Drilling costs   V

(*f*)

A   B   C   F   D   E   G   V

(*g*)

B   F   C   G   A   E   D   H   J   V

P5.7
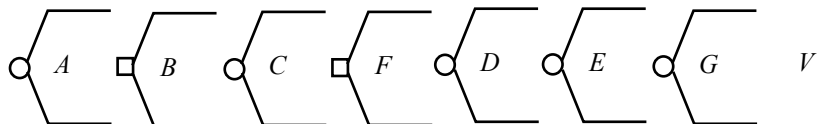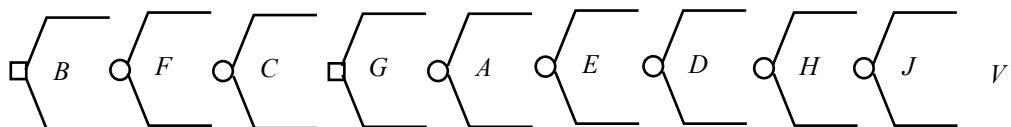
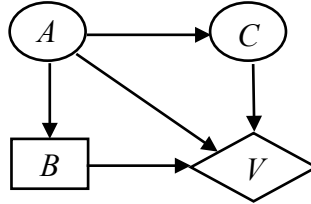**(*a*)** Observations:

- Info on A is available before decision B.
- C is dependent on A since $p(C_1|A_1) = 0.6 \neq p(C_1|A_2) = 0.5$
- C is independent of decision B given information on A:
  $$p(C_1|B_1,A_1) = p(C_1|B_2,A_1) = 0.6$$
  $$p(C_1|B_1,A_2) = p(C_1|B_2,A_2) = 0.5$$
- Value is dependent on all A, B, C (all the numbers are different).
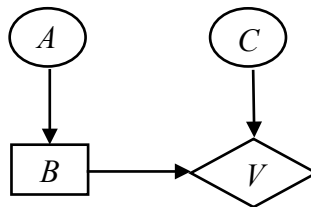
The influence diagram:

(*b*) Observations:

- Info on A is available before decision B.
- C is independent of A and B
  $$p(C_1|A_1) = p(C_1|A_2) = 0.6$$
  $$p(C_1|B_1) = p(C_1|B_2) = 0.6$$
- Value is independent of A.
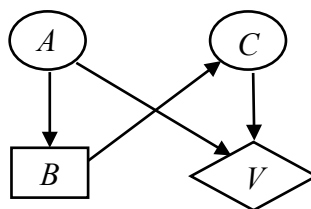
The influence diagram:

(*c*) Observations:

- Info on A is available before decision B.
- C is dependent on decision B
  $$p(C_1|B_1) = 0.6 \neq p(C_1|B_2) = 0.9$$
- C is independent of A given B
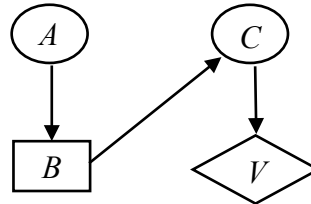 $$p(C_1|A_1,B_1) = p(C_1|A_2,B_1) = 0.6$$
  $$p(C_1|A_1,B_2) = p(C_1|A_2,B_2) = 0.9$$
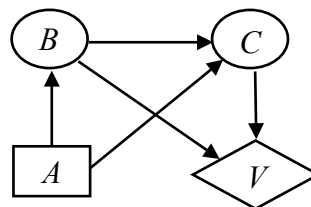- Value is independent of B.

The influence diagram:

(*d*)   Observations:
- Info on A is available before decision B.
- C is dependent on decision B
$$p(C_1|B_1) = 0.6 \neq p(C_1|B_2) = 0.9$$
- C is independent of A given B
$$p(C_1|A_1,B_1) = p(C_1|A_2,B_1) = 0.6$$
$$p(C_1|A_1,B_2) = p(C_1|A_2,B_2) = 0.9$$
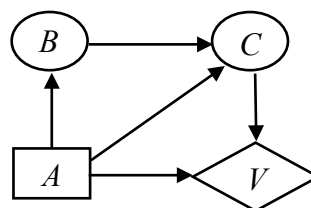- Value is independent of A and B.

The influence diagram:

(*e*)   Observations:
- B is dependent on decision A
$$p(B_1|A_1) = 0.2 \neq p(B_1|A_2) = 0.1$$
- C is dependent on B
$$p(C_1|B_1,A_1) = 0.6 \neq p(C_1|B_2,A_1) = 0.1$$
- C is dependent on A
$$p(C_1|A_1) = (.2)(.6)+(.8)(.1) = 0.2 \neq \quad p(C_1|A_2) = (.1)(.15)+(.9)(.65) = 0.6$$
- Value is independent of A.

The influence diagram:

(*f*)   Observations:
- B is dependent on decision A
$$p(B_1|A_1) = 0.2 \neq p(B_1|A_2) = 0.1$$
- C is dependent on B
$$p(C_1|B_1,A_1) = 0.6 \neq p(C_1|B_2,A_1) = 0.1$$
- C is dependent on A
$$p(C_1|A_1) = (.2)(.6)+(.8)(.1) = 0.2 \neq \quad p(C_1|A_2) = (.1)(.5)+(.9)(.75) = 0.725$$
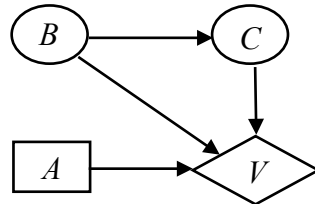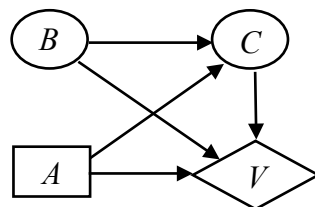- Value is independent of B.

The influence diagram:

(*g*)   Observations:

- *B* is independent of decision *A*
  $$p(B_1|A_1) = 0.2 = p(B_1|A_2)$$
- *C* is independent of decision *A*
  $$p(C_1|A_1) = (.2)(.6)+(.8)(.1) = 0.2$$
  $$p(C_1|A_2) = (.2)(.5)+(.8)(.125) = 0.2$$
- *C* is dependent on *B*
  $$p(C_1|B_1,A_1) = 0.6 \neq p(C_1|B_2,A_1) = 0.1$$
- Value is dependent on all *A*, *B*, *C*.

- Based on the above computations, the most compact influence diagram should be:

- However, the above diagram requires $p(C \mid B)$ which cannot be obtained from the decision tree because *A* is a decision node. $p(C \mid B)$ can only be obtained by a direct assessment between *B* and *C* either by the expert or by using the data. If such information is not available, then the following diagram may be drawn using probabilities directly available from the tree. But in so doing, we lose the fact that *C* is also independent of *A*.
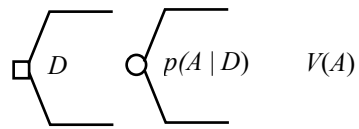

(*h*)   Observations:

- *B* is independent of decision *A*
  $$p(B_1|A_1) = 0.2 = p(B_1|A_2)$$
- C is dependent on decision A
  $$p(C_1|A_1) = (.2)(.6)+(.8)(.6) = 0.6 \neq$$
  $$p(C_1|A_2) = (.2)(.7)+(.8)(.7) = 0.7$$
- *C* is independent of *B* given *A*
  $$p(C_1|B_1,A_1) = 0.6 = p(C_1|B_2,A_1)$$
  $$p(C_1|B_1,A_2) = 0.7 = p(C_1|B_2,A_2)$$
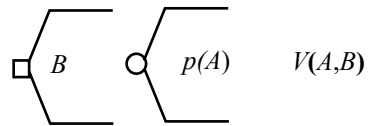- Value is independent of *B*.
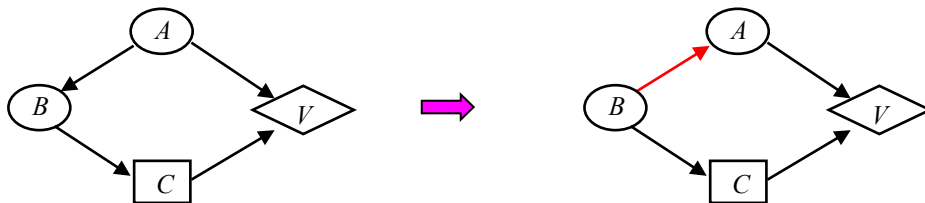
The influence diagram:
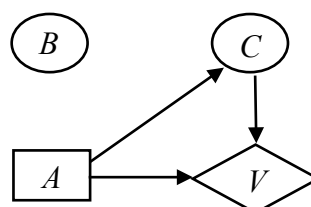
P5.8   (Optional Problem)
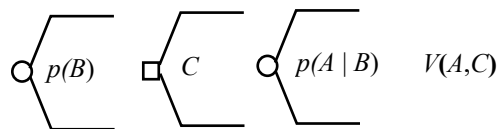
(*a*)   Tree Sequence = {*D, A, V*}

D $p(A \mid D)$ $V(A)$

(*b*)   Tree Sequence = {*B, A, V*}

B $p(A)$ $V(A,B)$

(*c*)   Convert to decision tree network:
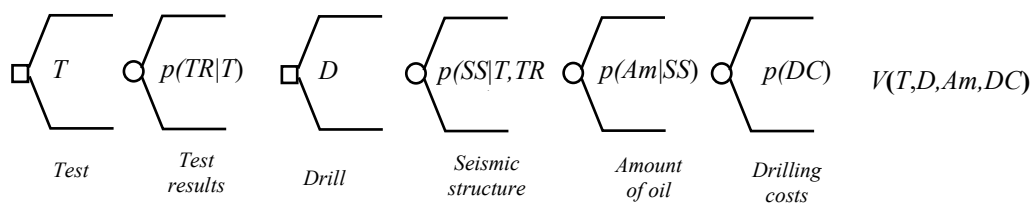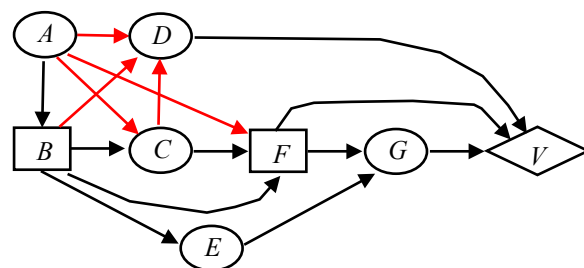

Tree Sequence = {*B, C, A, V*}

$p(B)$ C $p(A \mid B)$ $V(A,C)$

(*d*)   Tree Sequence = {*B, A, C, V*}
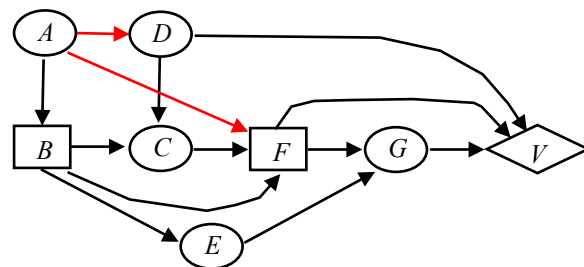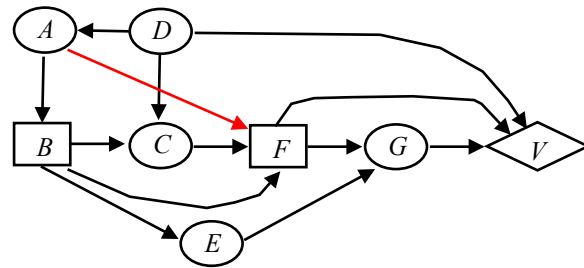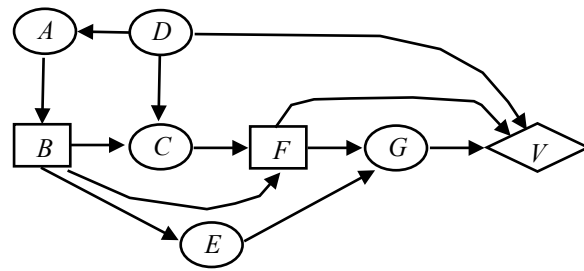

B $p(A)$ C $V(B,C)$
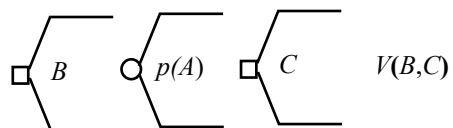
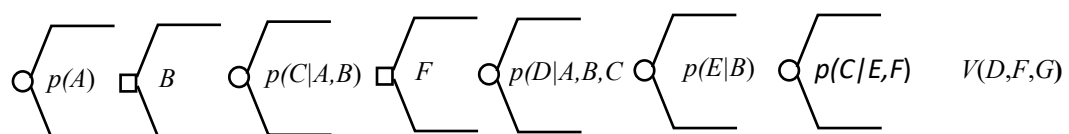**(*e*)**   Convert to decision tree network:
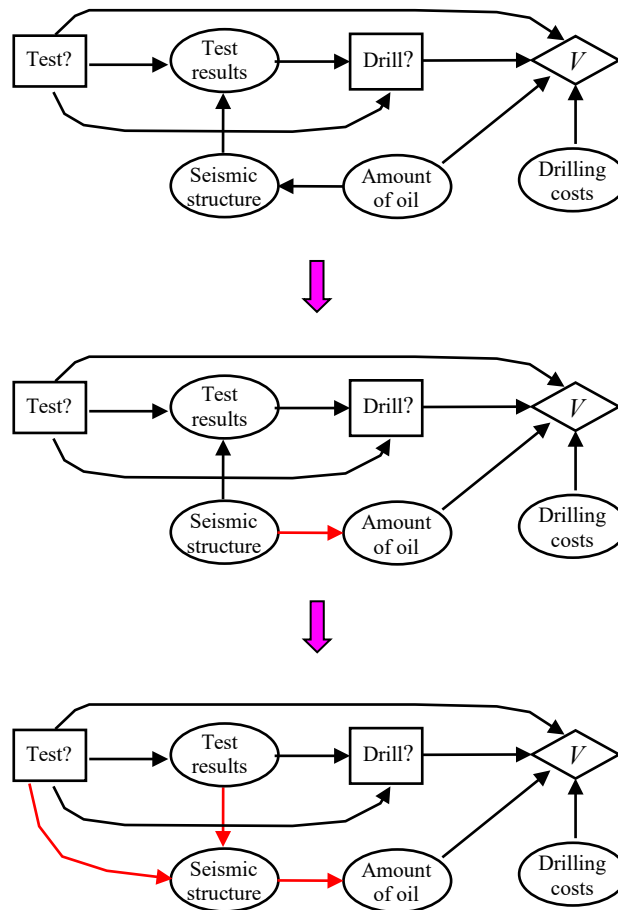
Sequence = {Test, Test results, Drill, Seismic structure, Amount of oil, Drilling costs, V }

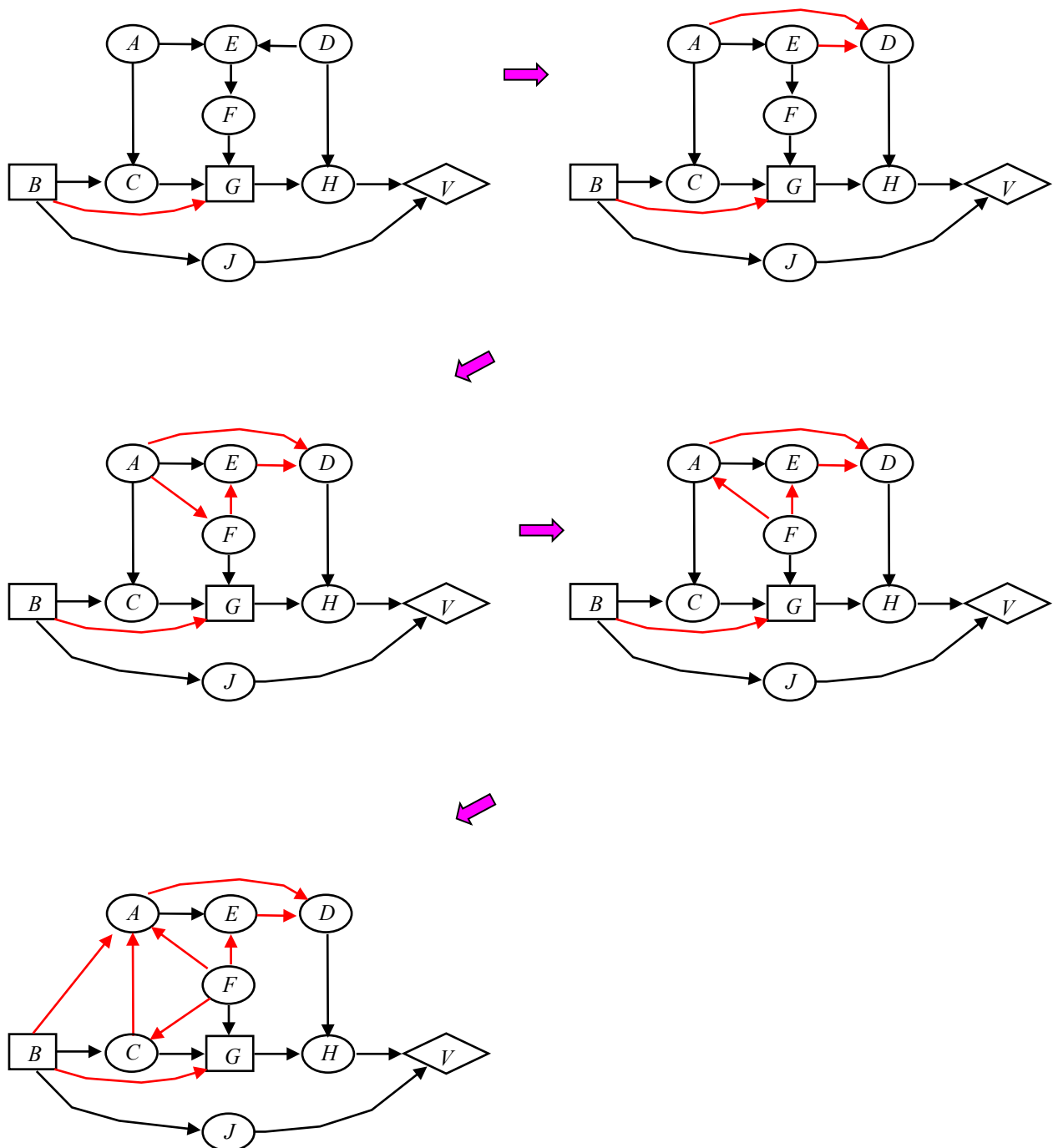(*f*)   Convert to decision tree network:

Tree Sequence = {*A*, *B*, *C*, *F*, *D*, *E*, *G*, *V* }

(g) Convert to decision tree network:

Sequence = { B, F, C, G, A, E, D, H, J ,V }

B   p(F)   p(C|B,F)   G   p(A|B,C,F)   p(E|A,F)   p(D|A,E)   p(H|G,D)   p(J|B)   V(H,J)