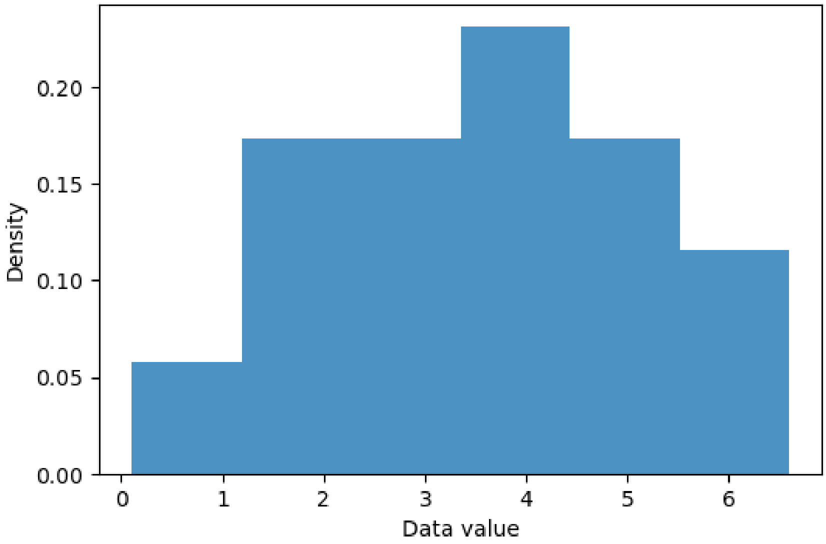
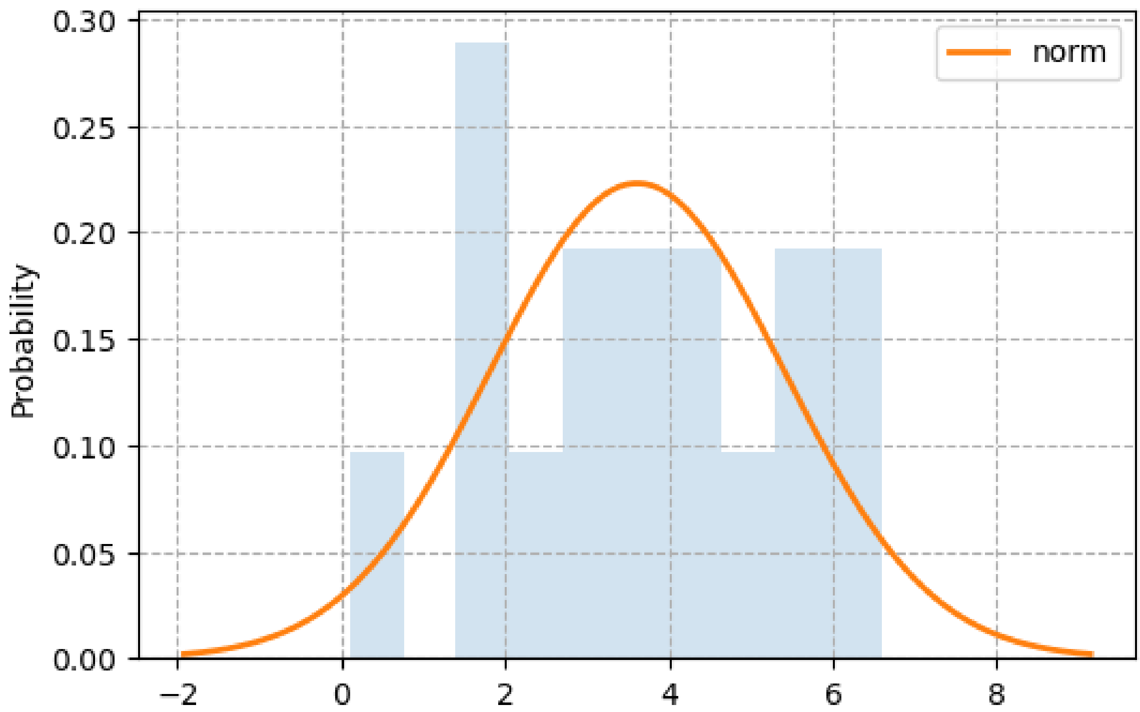
IE5203 Decision Analysis
Solutions to Chapter 7 Exercises

P7.1



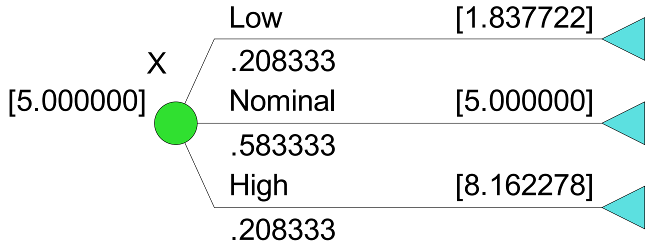
Data Description:
  size = 16
  minmax = (0.1077, 6.5941)
  mean = 3.61086875
  var = 3.41809
  std dev = 1.7901

The Maximum Likelihood Estimators (MLE) for the mean and standard deviation of the Normal Distribution are the mean and standard deviation of the observed data.
Hence, we will fit a Normal distribution with mean = 3.611 and standard deviation = 1.790

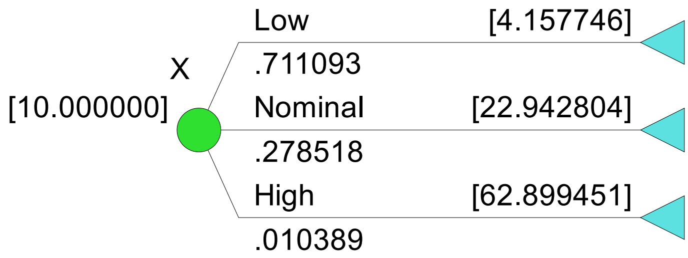


P7.2

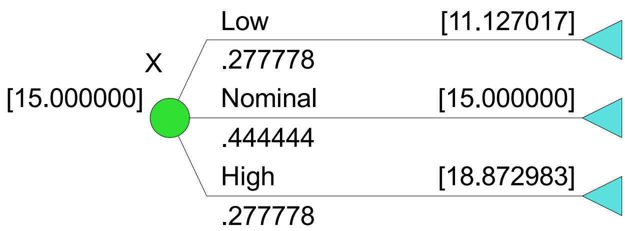
(***a***) Triangular (0, 10, 5)



(***b***) Exponential (1/10)



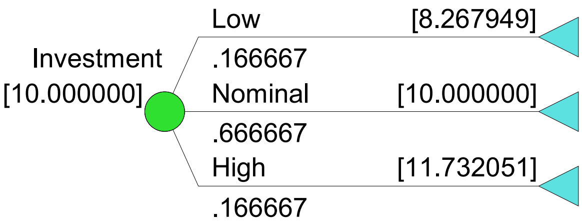
(***c***) Uniform (10, 20)



P7.3

Let the utility function be $u(x) = 1 - e^{-x/5}$ where x is in millions of dollars.

(*a*) Discrete 3-branch approximation (moments matching) using DPL:



$E[u(x)] = (1/6)\, u(8.267849) + (2/3)\, u(10) + (1/6)\, u(11.732051) = 0.861931$

$CE = u^{-1}(0.861931) = \$\ 9.9000$ millions

(*b*) Stanford/SDG 3-branch quick approximation:

From the CDF, the 10th, 50th, and 90th percentiles are 8.718, 10.0, and 11.282, respectively.

$E[u(x)] = 0.25\, u(8.718) + 0.5\, u(10.0) + 0.25\, u(11.282) = 0.86243$

$CE = u^{-1}(0.86243) = \$\ 9.9181$ millions

(*c*) Pearson-Tukey 3-branch approximation method:

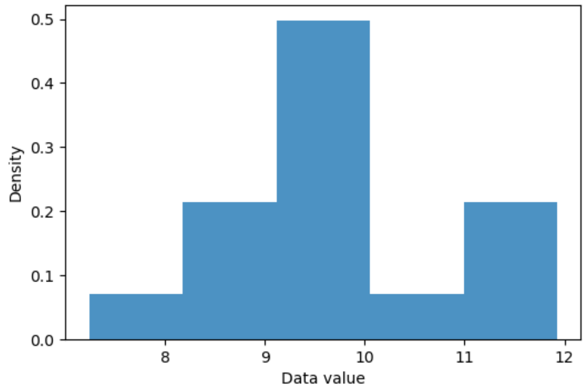
From the CDF, the 5th, 50th, and 95th percentiles are 8.355, 10.0, and 11.645, respectively.

$E[u(x)] = 0.185\, u(8.355) + 0.63\, u(10.0) + 0.185\, u(11.645) = 0.86193$

$CE = u^{-1}(0.86193) = \$\ 9.9000$ millions

P7.4

(*a*) The data:



Data Description:
size = 15
minmax = (7.24, 11.93)
mean = 9.762
var = 1.4419457142857142

The top 5 fitted distributions based on KS:

Distributions: laplace
Parameters = (9.6200, 0.8300)
KS statistic = 0.14891708541566395
KS p-value = 0.8464890809943011
mean = 9.6200
var = 1.3778
std dev = 1.1738

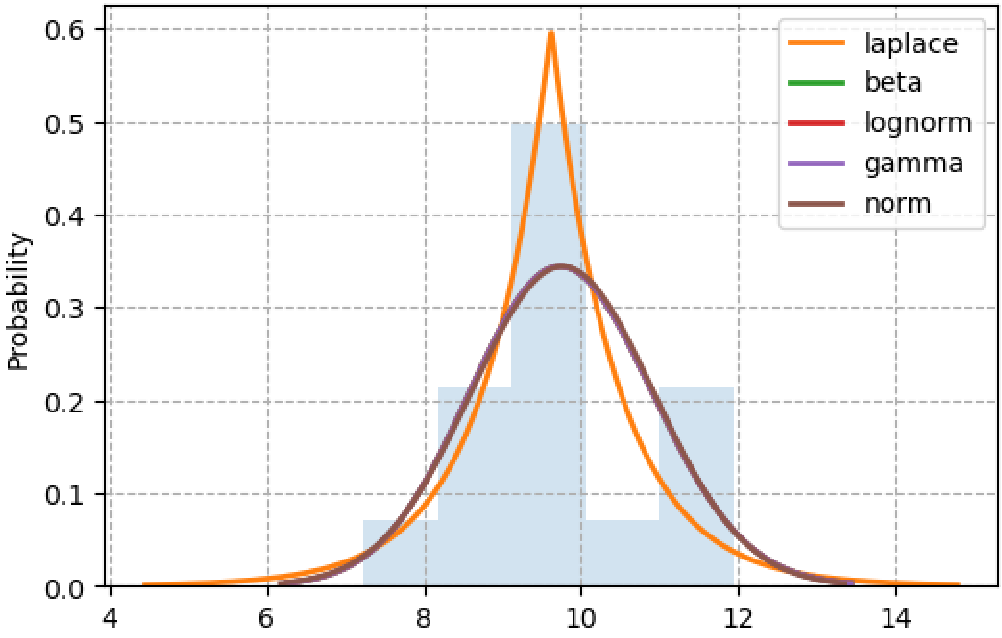
Distribution: beta
Parameters = (1410.7634, 58247502.1299, -33.8063, 1798873.4614)
KS statistic = 0.1960370649094899
KS p-value = 0.5471048303675403
mean = 9.7616
var = 1.3455
std dev = 1.1599

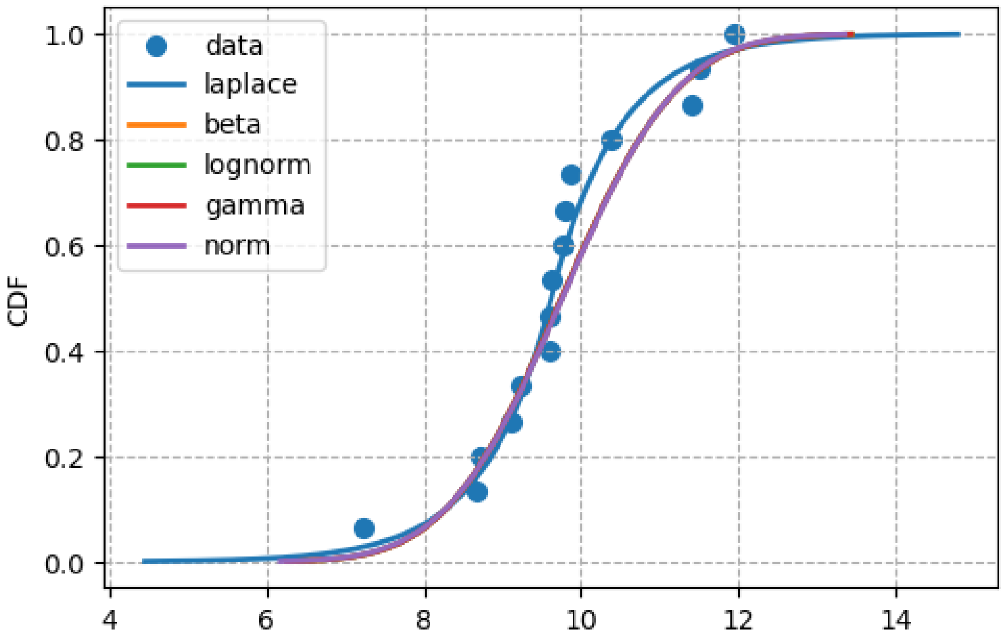
Distribution: lognorm
Parameters = (0.0171, -58.1049, 67.8569)
KS statistic = 0.19629155291299982
KS p-value = 0.5454833079669202
mean = 9.7620
var = 1.3458
std dev = 1.1601

Distribution: gamma
Parameters = (1555.1992, -35.9871, 0.0294)
KS statistic = 0.19633294518248245
KS p-value = 0.5452197271626447
mean = 9.7620
var = 1.3458
std dev = 1.1601

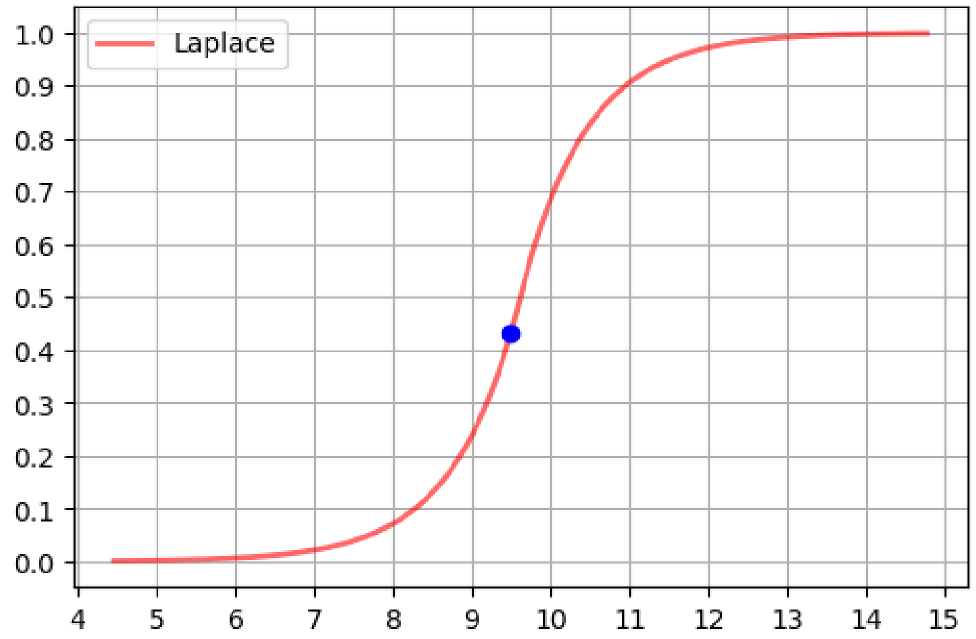
Distribution: norm
Parameters = (9.7620, 1.1601)
KS statistic = 0.19967233371824233
KS p-value = 0.5241052040775288
mean = 9.7620
var = 1.3458
std dev = 1.1601

(*b*) Comparing the PDF and CDF of the fitted distribution with the data.





(*c*) **CDF of the fitted distribution:**



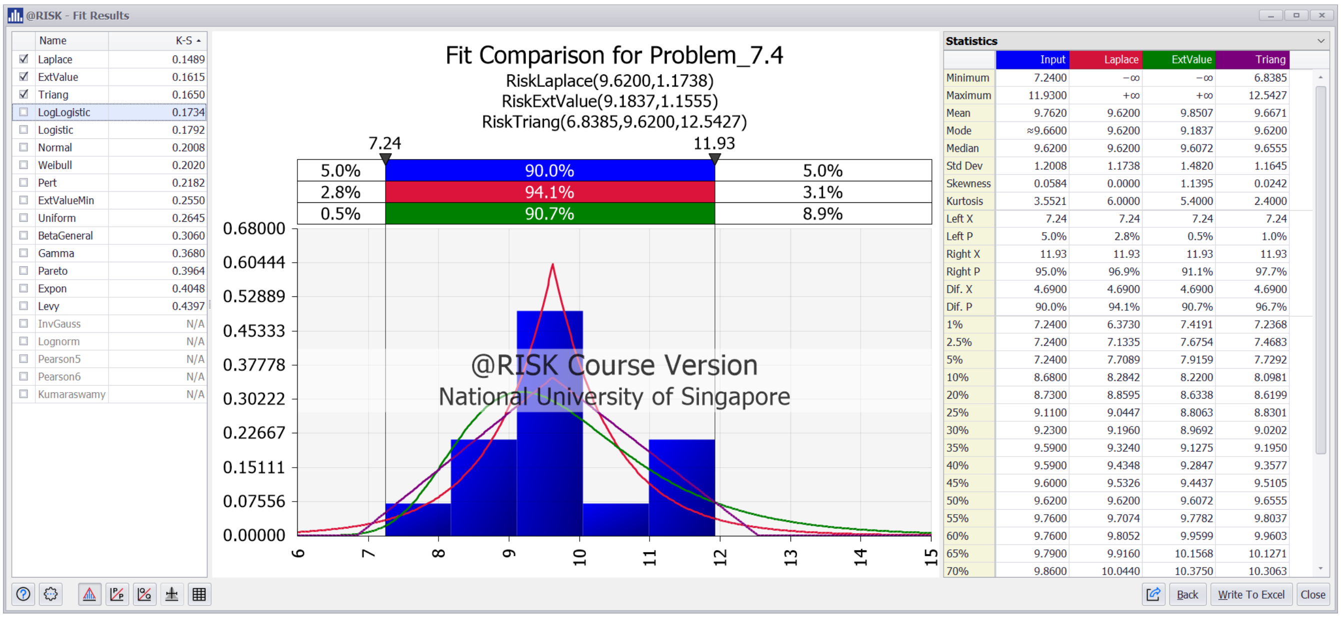
$$F(x) = \frac{1}{2} + \frac{1}{2}\ \mathrm{sgn}(x - \mu)\left(1 - \exp\left(-\frac{|x - \mu|}{b}\right)\right)$$

where $\mu = 9.6200$, $b = 0.8300$

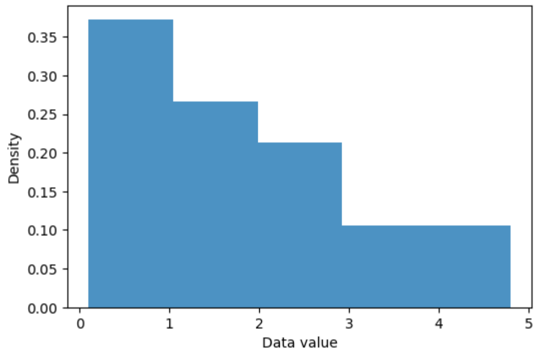
The probability that an animal weights less than 9.5 gram = **<u>0.4327</u>**

```
Python: scipy.stats.lapalce.cdf(9.5, 9.6200, 0.8300)
```

Using @Risk, we also get Laplace as the best fit distribution:

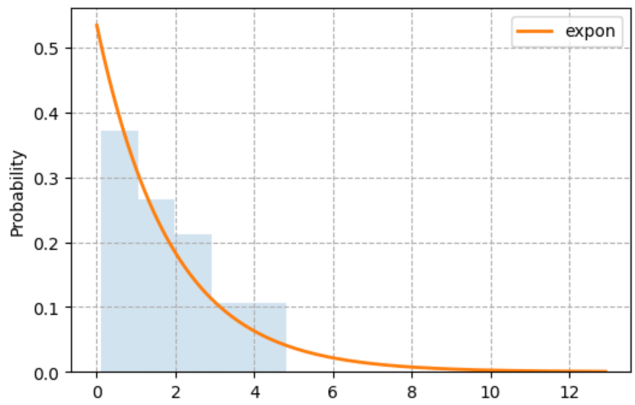


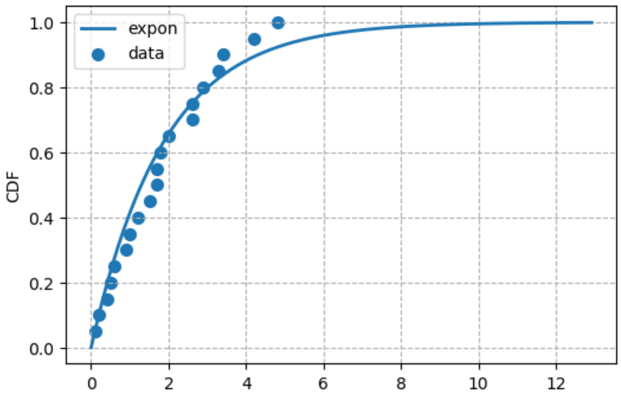
P7.5



Data Description:
size = 20
minmax = (0.1, 4.8)
mean = 1.8699999999999999
var = 1.8137894736842104
skewness = 0.5802772894968783
kurtosis= -0.5649727424575377

We will fit an exponential distribution with one parameter (location = 0).

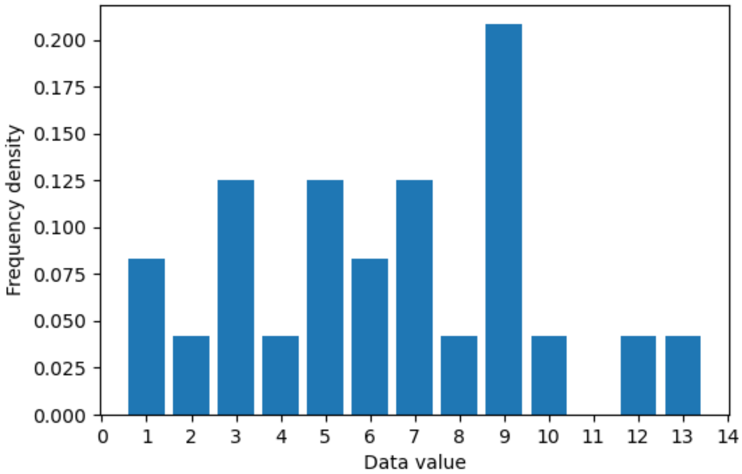




Distribution: expon
Parameters = (0.0000, 1.8700)
KS statistic = 0.15163114014127343
KS p-value = 0.6921603208893784
mean = 1.8700
var = 3.4969
std dev = 1.8700

P7.6

(*a*) Histogram of the data



Data Description:
 size = 24
 minmax = (1, 13)
 mean = 6.375
 var = 10.853260869565217
 skewness = 0.1004975735577767
 kurtosis= -0.7612716019447299

(*b*) Using DecisionAnalysisPy. DistFit_discrete Class

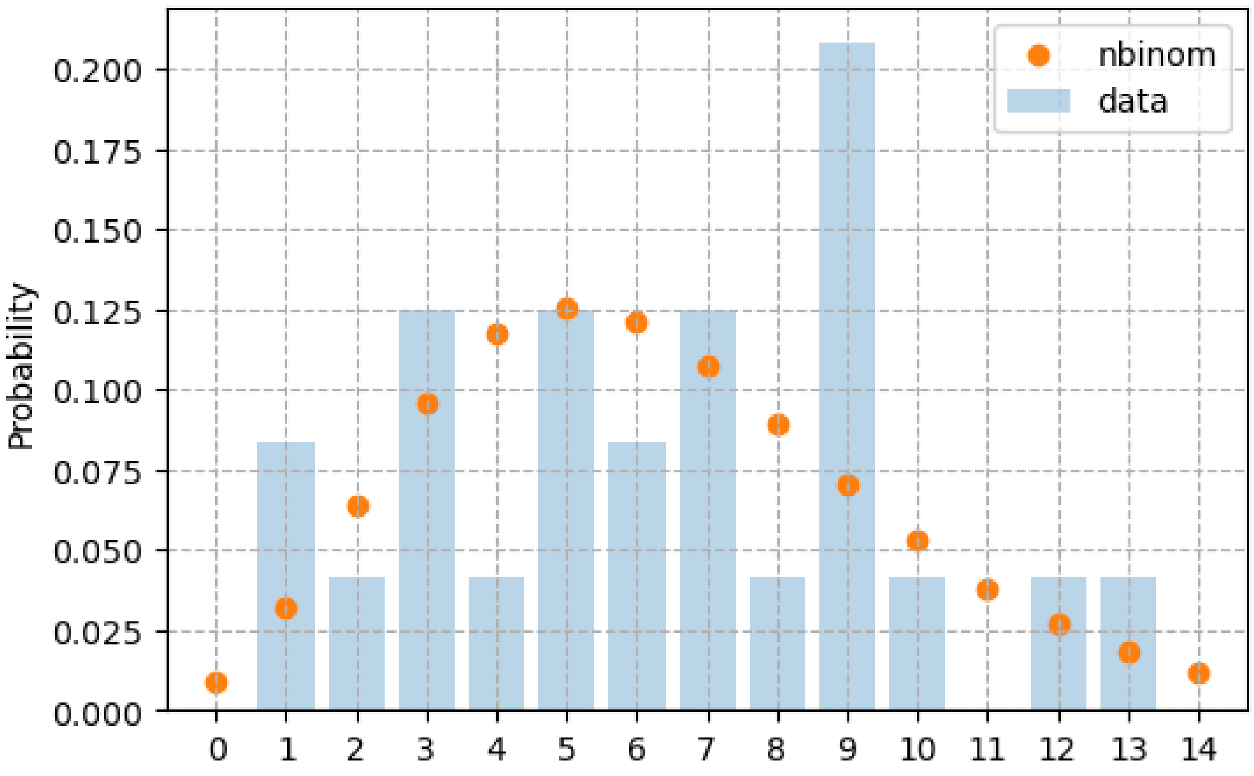
Top 3 discrete distribution fitted are:

```
Distribution 1: nbinom
  Params = [8.30298378 0.56567604]
  KS_stats = 0.17272838916584243
  p-value = 0.4233763247639363
  mean = 6.375000043564911
  var = 11.269701352459267
  sd = 3.3570375857978214

Distribution 2: poisson
  Params = [6.37500004]
  KS_stats = 0.2211842014782398
  p-value = 0.16391064587178117
  mean = 6.375000035762788
  var = 6.375000035762788
  sd = 2.5248762416726067

Distribution 3: binom
  Params = [14.   0.5]
  KS_stats = 0.24355061848958337
  p-value = 0.09726881318566716
  mean = 7.0
  var = 3.5
  sd = 1.8708286933869707
```

Distribution selected:

- negative binomial with n = 8.30298378 and p = 0.56567604]

(*c*) Comparing the PMF of the fitted distributions with the data



Using @Risk, we get the same distribution with parameters close.