# TIE2140 Engineering Economy
# Solutions to Tutorial #5

**Question 1.**

(*a*)

| | Pessimistic | Most likely | Optimistic |
|---|---|---|---|
| Capital Investment | $ 120,000 | $ 100,000 | $ 90,000 |
| Useful Life | 6 years | 10 years | 12 years |
| Market Value at EoL | $ 0 | $ 20,000 | $ 30,000 |
| Net annual cash flow | $ 20,000 | $ 30,000 | $ 35,000 |

*MARR* = 10%

***i.*** When all the factors are at their most likely values:

$AW(10\%) = -100{,}000\,[A/P,10\%,10] + 30{,}000 + 20{,}000\,[A/F,10\%,10] = \$\,14{,}980.37 > 0$

***ii.*** When all the factors are at their optimistic values:

$AW(10\%) = -90{,}000\,[A/P,10\%,12] + 35{,}000 + 30{,}000\,[A/F,11\%,12] = \$\,23{,}194.20 > 0$

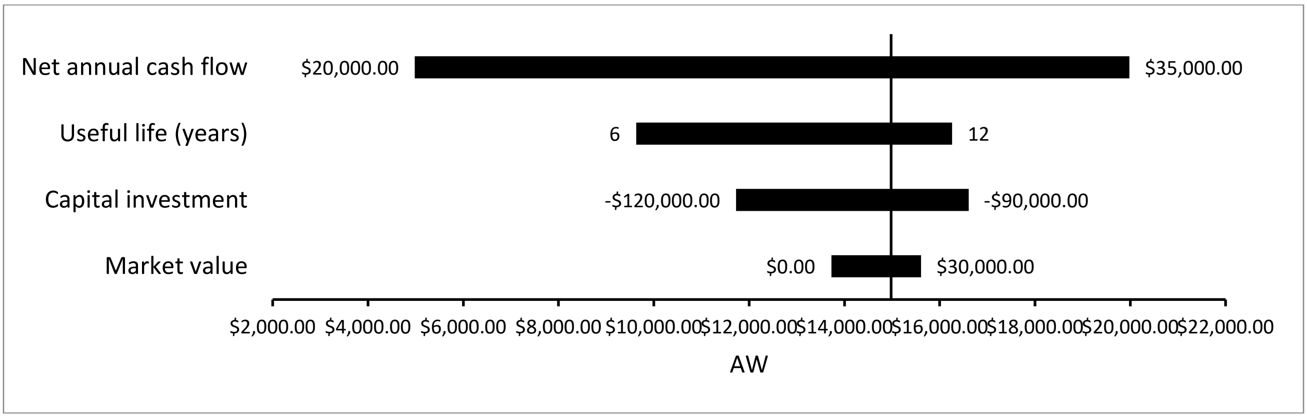
***iii.*** When all the factors are at their pessimistic values:

$AW(10\%) = -120{,}000\,[A/P, 10\%,6] + 20{,}000 + 0 = -\$\,7{,}552.89 < 0$

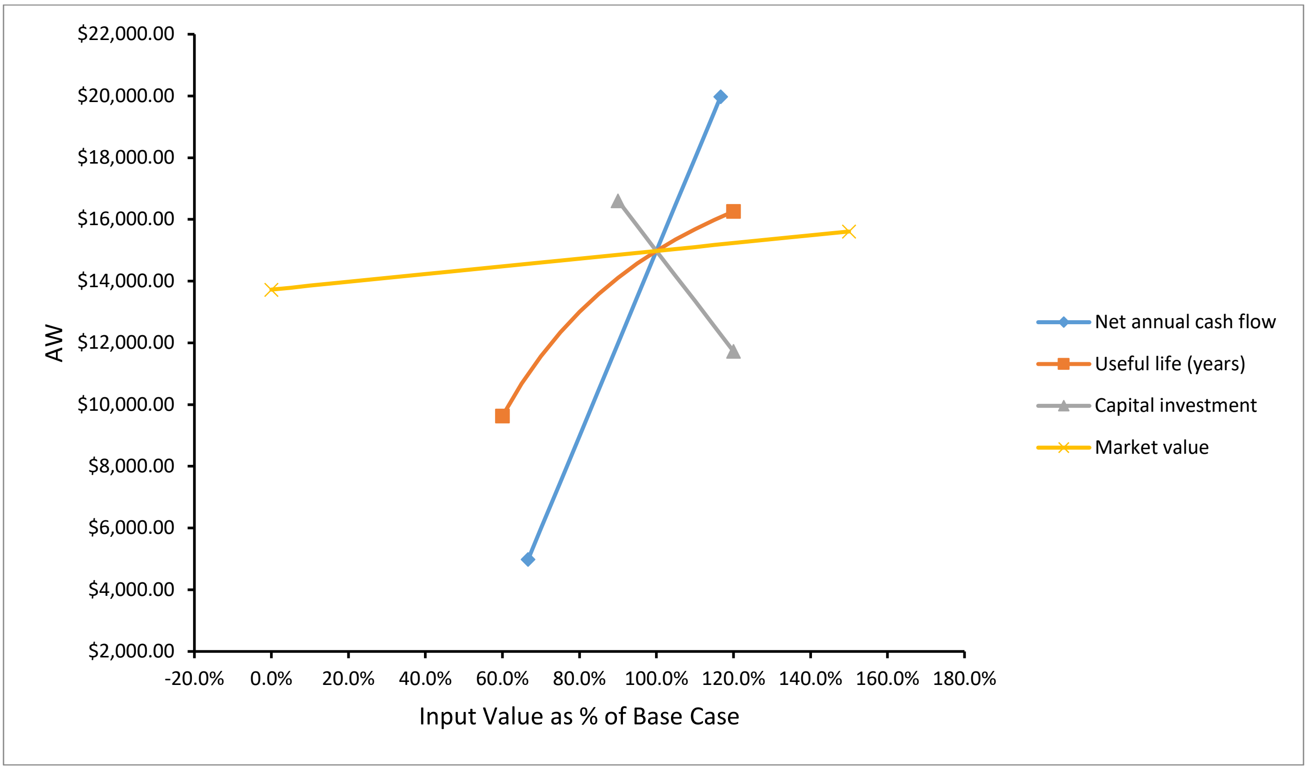
(*b*) **One-Way Range Sensitivity Table**:

| | | | | *AW* | | |
|---|---|---|---|---|---|---|
| **Variable** | **Pessimistic** | **Most-likely** | **Optimistic** | **Low** | **High** | **Swing** |
| Net annual cash flow | $ 20,000 | $ 30,000 | $ 35,000 | $4,980.37 | $19,980.37 | $15,000.00 |
| Useful life (years) | 6 | 10 | 12 | $9,631.41 | $16,258.93 | $6,627.52 |
| Capital investment | -$120,000 | -$100,000 | -$90,000 | $11,725.46 | $16,607.82 | $4,882.36 |
| Market value | $ 0 | $ 20,000 | $ 30,000 | $13,725.46 | $15,607.82 | $1,882.36 |

**Tornado Diagram for *AW***




**Spider Diagram for *AW***




**(*c*)** Identification of Sensitive and Non-sensitivity factors:

- The project *AW* is most sensitive to Net Annual Cash Flow, followed by Useful Life, and Capital Investment.

- Market Value at EoL is not very sensitive.

**Question 2.**

**(*a*)**

| | Winshear | Blowby | Air-vantage |
|---|---|---|---|
| Capital Investment | $1,000 | $400 | $1,200 |
| Drag reduction | 20% | 10% | 25% |
| Maintenance per year | $10 | $5 | $5 |
| Useful life | 10 years | 10 years | 5 years |

Study period = 10 years. Assume repeatability.

Let $X$ = the number of miles driven per year by a tractor.

**Reductions in fuel consumption for each alternative:**

- Windshear: (20 % / 5 %) (2 % per miles) = 8% per mile
- Blowby: (10 % / 5 %) (2 % per miles) = 4% per mile
- Air-vantage: (25 % / 5 %) (2 % per miles) = 10% per mile

**Annual Fuel Costs for each alternative:**

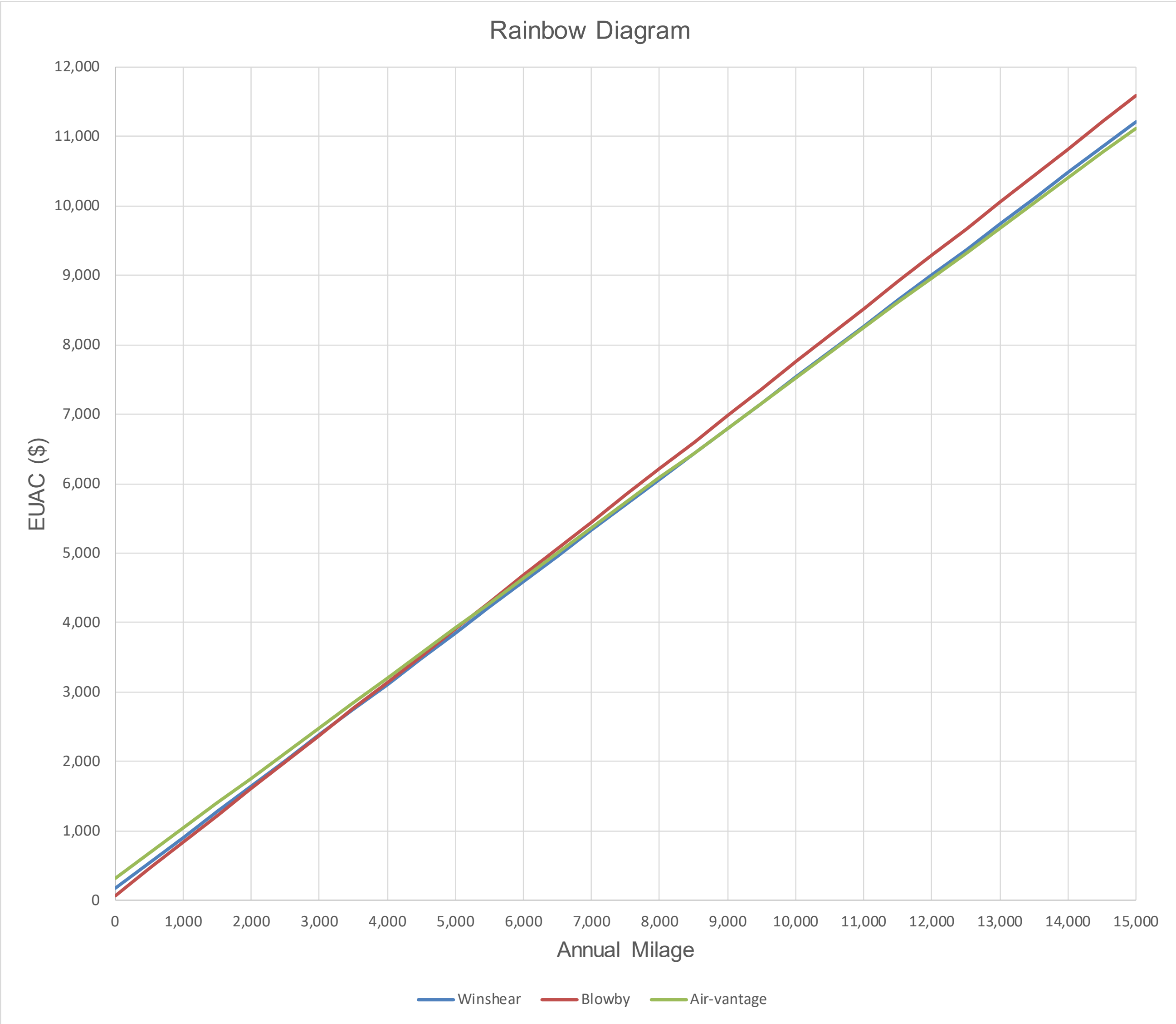
- Windshear: 0.92 $X$ (mile/year) × 1/5 (gallon/mile) × $4.00 ( per gallon) = $0.736$X$
- Blowby: 0.96 $X$ (mile/year) × 1/5 (gallon/mile) × $4.00 ( per gallon) = $0.768$X$
- Air-vantage: 0.90 $X$ (mile/year) × 1/5 (gallon/mile) × $4.00 ( per gallon) = $0.720$X$

**Equivalent Uniform Annual Cost (EUAC) for each alternative:**

- $EUAC$(Windshear) = 1,000 [$A/P$, 10%, 10] + 10 + 0.736 $X$
  = 1,000 (0.162745395) + 10 + 0.736 $X$
  = 172.74539 + 0.736 $X$

- $EUAC$(Blowby) = 400 [$A/P$, 10%, 10] + 5 + 0.768 $X$
  = 400 (0.162745395) + 5 + 0.768 $X$
  = 70.09816 + 0.768 $X$

- $EUAC$(Air-vantage)= 1,200 [$A/P$, 10%, 5] + 5 + 0.720 $X$
  = 1,200 (0.263797481) + 5 + 0.720 $X$
  = 321.55698 + 0.720 $X$




***(b)***

To determine the breakpoint value between Blowby and Windshear, we solve

$$
\begin{aligned}
EUAC(\text{Blowby}) &= EUAC(\text{Windshear}) \\
70.09816 + 0.768X &= 172.74539 + 0.736X \\
X &= 3{,}207.73
\end{aligned}
$$

To determine the breakpoint value between Windshear and Air-vantage, we solve

$$
\begin{aligned}
EUAC(\text{Windshear}) &= EUAC(\text{Air-vantage}) \\
172.74539 + 0.736X &= 321.55698 + 0.720X \\
X &= 9{,}300.73
\end{aligned}
$$

**Optimal Decision Rule**

| Miles driven per year | Optimal Choice |
|---|---|
| $0 < X \leq 3{,}207.73$ | Blowby |
| $3207.73 \leq X \leq 9{,}300.73$ | Windshear |
| $9{,}300.73 \leq X$ | Air-vantage |

**Question 3.**

| | **Alternative 1** | **Alternative 2** |
|---|---|---|
| Capital Investment | \$ 4,500 | \$ 6,000 |
| Annual revenues | \$ 1,600 | \$ 1,850 |
| Annual expenses | \$ 400 | \$ 500 |
| Estimated market value | \$ 800 | \$ 1,200 |
| Useful life | 8 years | 10 years |

*MARR* = 15%.

**(*a*)** Study period = 40 years. Assume repeatability.

*AW*(15%) of Alternative 1 over the 40-year study period
= *AW*(15%) of Alternative 1 over the first 8 years
= – 4,500 [*A*/*P*, 15%, 8] + 1,600 – 400 + 800 [*A*/*F*, 15%,8]
**= \$ <u>255.45</u>**

*AW*(15%) of Alternative 2 over 40-year study period
= *AW*(15%) of Alternative 2 over the first 10 years
= – 6,000 [*A*/*P*, 15%, 10] + 1,850 – 500 + 1,200 [*A*/*F*, 15%, 10]
**= \$ <u>213.59</u>**

Select **<u>Alternative 1</u>** with higher *AW* over study period 40 years.

***(b)***

Let $I_2$ = Capital cost of Alternative 2.

$$AW_2(15\%, I_2) = -I_2\,[A/P, 15\%, 10] + 1{,}850 - 500 + 1{,}200\,[A/F, 15\%, 10]$$

We want to find the value of $I_2$ such that $AW_2(15\%, , I_2) = AW_1(15\%)$:

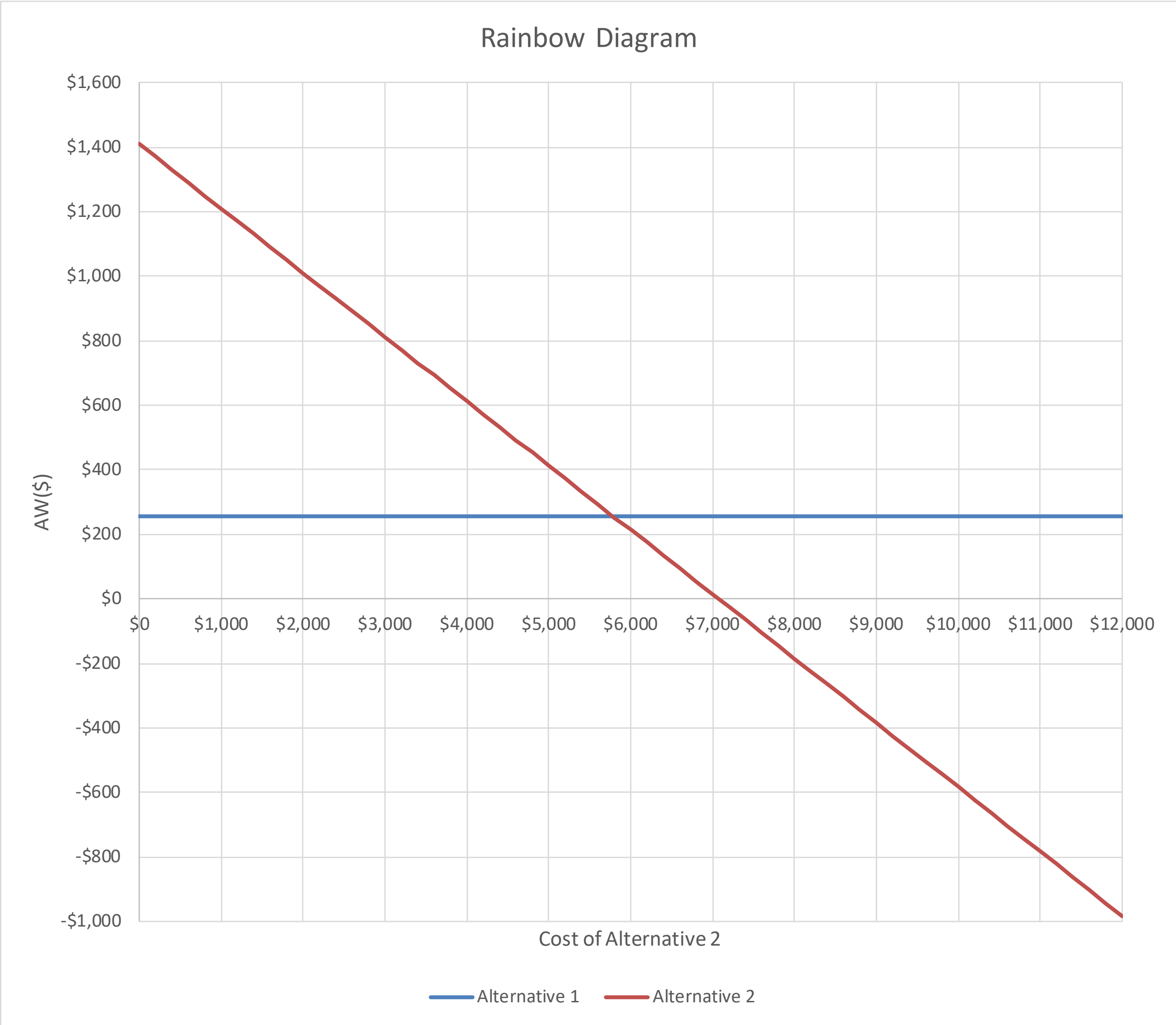
$$-\ I_2\,[A/P, 15\%, 10] + 1{,}850 - 500 + 1{,}200\,[A/F, 15\%, 10] \ = \ 255.45$$

$$\Rightarrow \qquad I_2 \ = \ \$\ 5{,}789.89$$

Change in capital cost of Alternative 2 required for decision reversal = 5,789.89 – 6,000

= – $ **<u>210.11</u>**

%-change required = – 210.11 / 6,000 = **– <u>3.502</u>%**



**(c)**

Let life of Alternative 1.

$AW_1(15\%, N_1) = -4{,}500\ [A/P, 15\%, N_1] + 1{,}600 - 400 + 800\ [A/F, 15\%, N_1]$

We want to find the value of $N_1$ such that $AW_1(15\%, N_1) = AW_2(15\%)$:

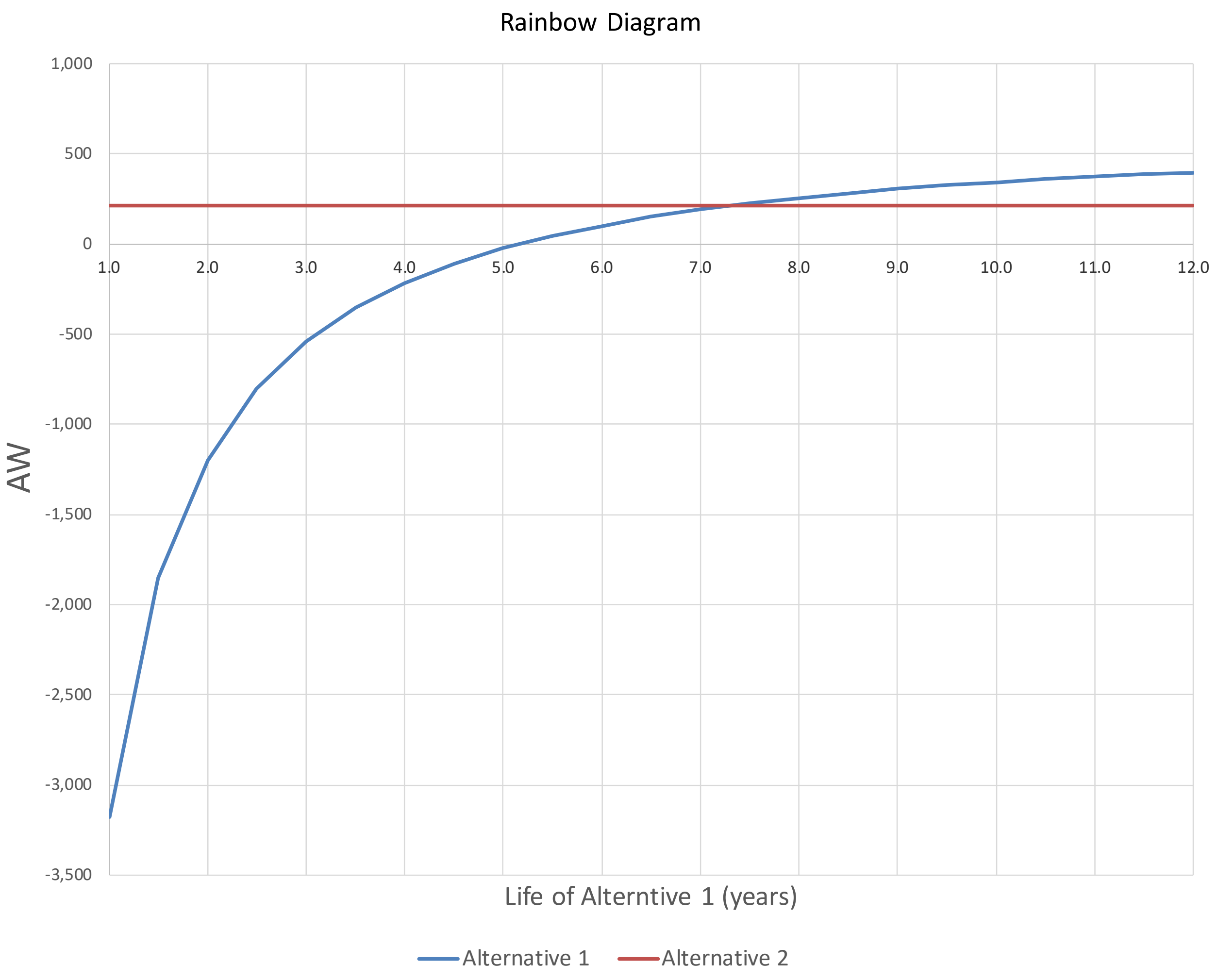
$$-4{,}500\ [A/P, 15\%, N_1] + 1{,}600 - 400 + 800\ [A/F, 15\%, N_1] = 213.59$$
$$-4{,}500\ [A/P, 15\%, N_1] + 986.41 \quad + 800\ [A/F, 15\%, N_1] = 0$$

Using Excel NPER function

$N_1$ =NPER(0.15, 986.41, -4500, 800, 0) = 7.321 years

Change in useful life of Alternative 1 required for decision reversal = 7.321 – 8 = – 0.679 years

%-change required = – 0.679 / 8 = **– 8.48%**



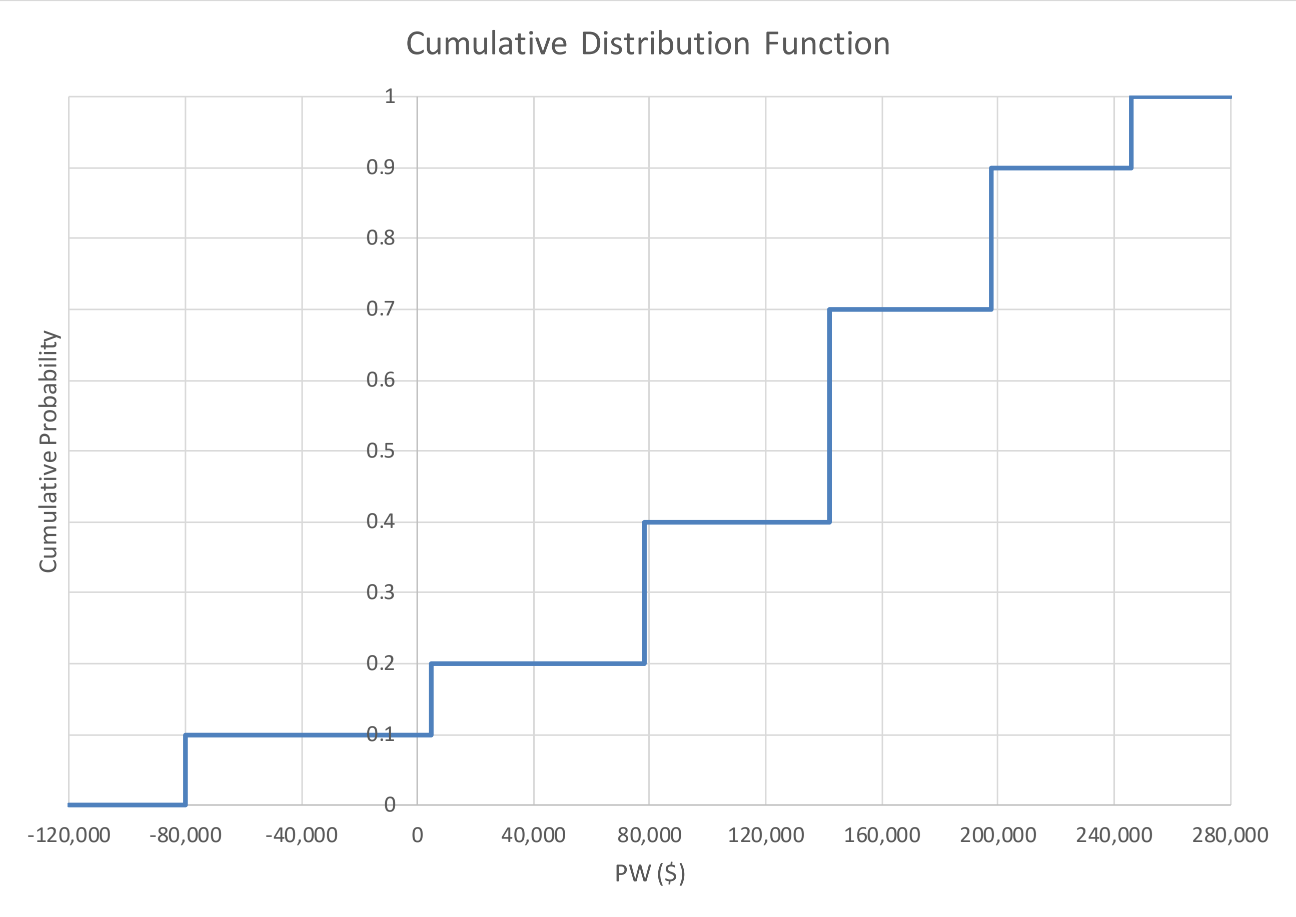
**Question 4.**

**(*a*)** $PW(N) = -500{,}000 + 200{,}000\ [P/A, 12\%, N]$

| $N$ | $p$ | $PW$ | $p \times PW$ | $PW - E[PW]$ | $p \times (PW - E[PW])^2$ |
|---|---|---|---|---|---|
| 3 | 0.1 | -80,082.68 | -8,008.27 | -194,943.70 | 3,800,304,582.45 |
| 4 | 0.1 | 4,536.80 | 453.68 | -110,324.22 | 1,217,143,324.65 |
| 5 | 0.2 | 78,118.95 | 15,623.79 | -36,742.06 | 269,995,823.16 |
| 6 | 0.3 | 142,103.44 | 42,631.03 | 27,242.42 | 222,644,871.05 |
| 7 | 0.2 | 197,742.12 | 39,548.42 | 82,881.10 | 1,373,855,485.38 |
| 8 | 0.1 | 246,123.58 | 24,612.36 | 131,262.57 | 1,722,986,141.40 |
| | | | 114,861.02 | | 8,606,930,228.10 |

$E[PW]$ = \$ 114,861.02
$Var[PW]$ = \$\$ 8,606,930,228.10
$\sigma[PW]$ = \$ 92,773.54

**(*b*)** The CDF for the $PW$ is plotted below:




**(*c*)**

***i.*** Downside Risk of Project = Prob$\{PW < 0\}$ = **0.1**

***ii.*** Chance of achieving an upside potential of $PW \geq \$150{,}000$ = 1 – 0.7 = **0.3**

***iii.*** Present Equivalent Value-at-Risk (95% confidence) = – (–80,082.68) = **\$ 80,082.68**

**Question 5.**

$MARR = 15\%$.

| | Alternative *A* | | Alternative *B* | |
|---|---|---|---|---|
| EoY | Expected Cash Flow ($) | Std Dev. of Cash Flow ($) | Expected Cash Flow ($) | Std Dev. of Cash Flow ($) |
| 0 | -8,000 | 0 | -12,000 | 500 |
| 1 | 4,000 | 600 | 4,500 | 300 |
| 2 | 6,000 | 600 | 4,500 | 300 |
| 3 | 4,000 | 800 | 4,500 | 300 |
| 4 | 6,000 | 800 | 4,500 | 300 |

**(*a*)** When the cash flows are mutually independent:

For Investment *A*:

$$E[PW(A)] = -8{,}000 + \frac{4{,}000}{(1+0.15)} + \frac{6{,}000}{(1+0.15)^2} + \frac{4{,}000}{(1+0.15)^3} + \frac{6{,}000}{(1+0.15)^4}$$
$$= \$6{,}075.71$$

$$Var[PW(A)] = 0 + \frac{600^2}{(1+0.15)^2} + \frac{600^2}{(1+0.15)^4} + \frac{800^2}{(1+0.15)^6} + \frac{800^2}{(1+0.15)^8}$$
$$= \$\$\ 963{,}949.69$$

$$\sigma[PW(A)] = \sqrt{963{,}949.69}$$
$$= \$\,981.81$$

For Investment *B*:

$$E[PW(B)] = -12{,}000 + \frac{4{,}500}{(1+0.15)} + \frac{4{,}500}{(1+0.15)^2} + \frac{4{,}500}{(1+0.15)^3} + \frac{4{,}500}{(1+0.15)^4}$$
$$= \$847.40$$

$$Var[PW(B)] = 500^2 + \frac{300^2}{(1+0.15)^2} + \frac{300^2}{(1+0.15)^4} + \frac{300^2}{(1+0.15)^6} + \frac{300^2}{(1+0.15)^8}$$
$$= \$\$\ 437{,}841.37$$

$$\sigma[PW(B)] = \sqrt{437{,}841.37}$$
$$= \$\,661.70$$

Investment *A* has a higher expected *PW* than Investment *B*, but the standard deviation of Invstment *A* is larger than that of Investment *B*. Hence the mean-variance criterion is non conclusive.

**(*b*)** When the cash flows are mutually independent;

$$E[PW(A-B)] = E[PW(A)] - E[PW(B)]$$
$$= 6{,}075.71 - 847.40$$
$$= \$\ 5{,}228.30$$

$$Var[PW(A-B)] = Var[PW(A)] + Var[PW(B)]$$
$$= 963{,}949.69 + 437{,}841.37$$
$$= \$\$\ 1{,}401{,}791.05$$

$$\sigma[PW(A-B)] = \sqrt{1{,}401{,}791.05}$$
$$= \$1{,}183.97$$

**(*c*)** When not all the cash flows for investment *A* are mutually independent and coefficients of correlations are $\rho_{12} = 0.1$, $\rho_{23} = 0.2$, $\rho_{34} = 0.3$:

$$E[PW(A)] = \$\ 6{,}075.71$$

$$Var[PW(A)] = 0 + \frac{600^2}{(1+0.15)^2} + \frac{600^2}{(1+0.15)^4} + \frac{800^2}{(1+0.15)^6} + \frac{800^2}{(1+0.15)^8}$$
$$+ \frac{2(0.1)(600)(600)}{(1+0.15)^3} + \frac{2(0.2)(600)(800)}{(1+0.15)^5} + \frac{2(0.3)(800)(800)}{(1+0.15)^7}$$
$$= \$\$1{,}251{,}108.61$$

$$\sigma[PW(A)] = \sqrt{1{,}251{,}108.61}$$
$$= \$1{,}118.53$$