TIE4203 Decision Analysis in Industrial & Operations Management
Solutions to Tutorial #6

Question 1 (P6.1)

(*a*) $u(w) = w - \beta w^2$

$u'(w) = 1 - 2\beta w$

$u''(w) = -2\beta$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = \dfrac{1-2\beta w}{2\beta}$

Degree of absolute risk aversion $r(w) = \dfrac{2\beta}{1-2\beta w}$

Note that for $u(w)$ to be increasing and concave (i.e., risk averse), it is sufficient for the coefficient β to be strictly positive.

(*b*) $u(w) = ln\ w$

$u'(w) = 1/w$

$u''(w) = -1/w^2$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = w$

Degree of absolute risk aversion = $r(w) = \dfrac{1}{w}$

(*c*) $u(w) = sgn(\beta)\ w^{\beta}$

$u'(w) = sgn(\beta)\ \beta\ w^{\beta-1}$

$u''(w) = sgn(\beta)\ \beta\ (\beta - 1)\ w^{\beta-2}$

Risk tolerance $\rho(w) = \dfrac{-u'(w)}{u''(w)} = \dfrac{w}{1-\beta}$

Degree of absolute risk aversion = $r(w) = \dfrac{1-\beta}{w}$

Question 2 (P6.2)

Given that John has the utility function $u(x) = 1 - 3^{-x/50}$ over the range of x = -\$50 to \$5,000.

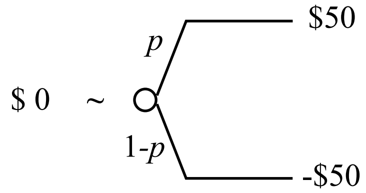
The utility function may be rewritten as $u(x) = 1 - 3^{-x/50} = 1 - e^{-x \ln 3/50}$

(*a*) The utility function is increasing and concave. Hence John is risk averse.

(*b*) John's risk tolerance = \$ 50 / *ln* 3.

Hence John's degree of risk aversion = 1 / risk tolerance = *ln* 3 / 50 = 0.0219722 $\$^{-1}$

(*c*) We want to find p such that the certainty equivalent of the deal is zero.

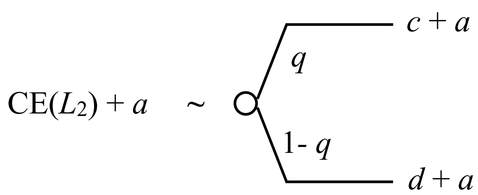


$$
\begin{aligned}
u(0) &= p\, u(50) + (1-p)\, u(-50) \\
1 - 3^0 &= p\,(1 - 3^{-1}) + (1-p)\,(1 - 3^1) \\
0 &= p\,(2/3) + (1-p)(-2) \\
p &= 3/4
\end{aligned}
$$

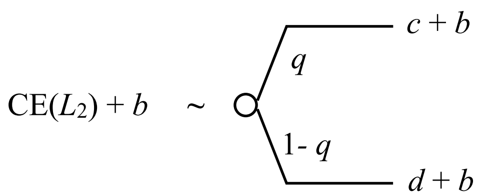
Question 3 (P6.3)

Consider L_2:

By the delta property, if we add the amount a to all its outcomes of L_2, we get

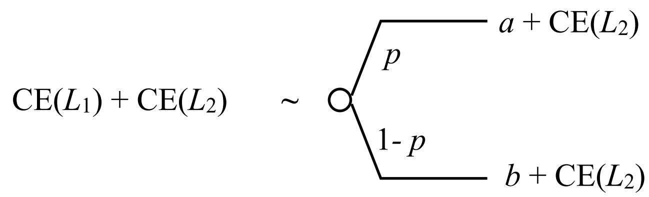


Similarly, by the delta property, if we add the amount b to all the outcomes of L_2, , we get



Consider L_1:

By the delta property, if we add the amount CE(L_2) to all its outcomes, we get



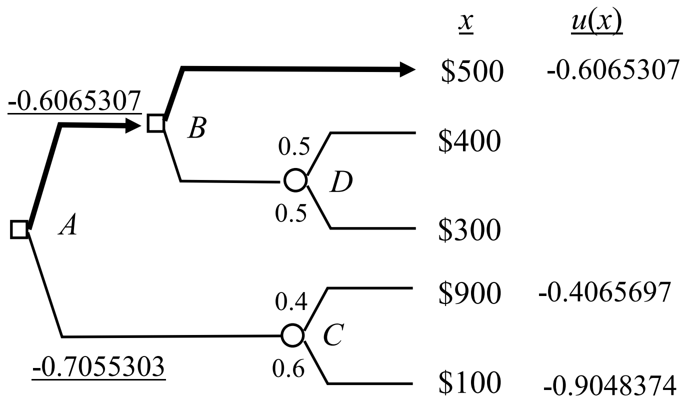
Finally, by the subsitution rule, the above deal is equivalent to L_3.

Hence CE(L_3) = CE(L_1) + CE(L_2).

Question 4 (P6.4)

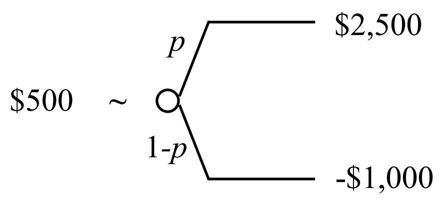
Since George has a constant risk tolerance of \$1,000, it follows that he has constant absolute risk aversion or the delta property. Hence his utility function is exponential in form.

We let $u(x) = -\exp(-x/1{,}000)$.



Maximum Expected Utility at A = -0.6065307
Certainty Equivalent at $A = u^{-1}(-0.60653)$ = \$500

The required preference probability is p, such that



Hence $\quad u(500) = p\, u(2{,}500) + (1-p)\, u(-1{,}000)$

$$\Rightarrow \quad -e^{\frac{-500}{1,000}} = (p)(-e^{\frac{-2,500}{1,000}}) + (1-p)(-e^{\frac{1,000}{1,000}})$$

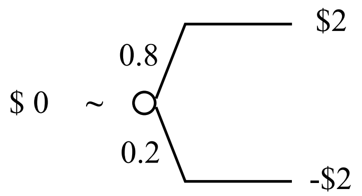
$$\Rightarrow \quad p = 0.80106$$

Note that we would have got the same answer if we had used the utility function $u(x) = 1 - exp(-x/1000)$, or if we had assumed $u(x) = a - b\ exp(-x/1000)$ and fitted the constants a and b to the boundary conditions $u(-\$1{,}000) = 0$ and $u(\$2{,}500) = 1$. Make sure you understand why this is so.

Question 5 (P6.5)

(*a*) Susan satisfies delta property $\Rightarrow$ utility function is of the form $u(x) = a - b\,e^{-x/\rho}$ where ρ is the risk tolerance.

Given



$$
\begin{aligned}
u(0) &= 0.8\,u(2) + 0.2\,u(-2) \\
a - b &= 0.8\,(a - b\,e^{-2/\rho}) + 0.2\,(a - b\,e^{2/\rho}) \\
1 &= 0.8\,e^{-2/\rho} + 0.2\,e^{2/\rho}
\end{aligned}
$$

Let $\quad x = e^{2/\rho}$

$$x^2 - 5x + 4 = 0 \;\Rightarrow\; (x-1)(x-4) = 0 \;\Rightarrow\; x = 1 \quad \text{or} \quad x = 4.$$

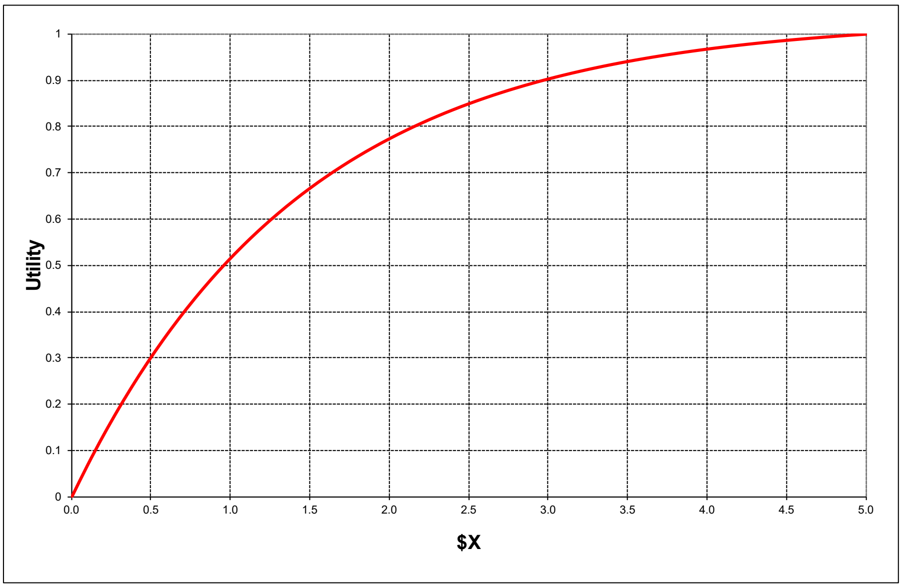
Hence $= e^{2/\rho} = 4 \;\Rightarrow\; \rho = \dfrac{1}{\ln 2} = \$1.44$

Susan risk tolerance = \$1.44

(*b*) Susan's risk attitude is risk adverse since her risk tolerance is positive.

(*c*) Susan's utility function such that $u(L = 0) = 0$ and $u(H = 5) = 1$ is

$$
\begin{aligned}
u(x) &= \frac{1 - e^{-(x-L)/\rho}}{1 - e^{-(H-L)/\rho}} \\
&= 1.0322581\ (1 - e^{-x\ln 2}) \\
&= 1.0322581\ (1 - 2^{-x})
\end{aligned}
$$

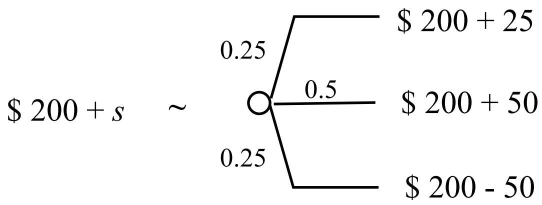


Question 6 (P6.6)

Current wealth = \$200.

Wealth Utility function $\quad u(w) = \dfrac{w^2}{2000},\ w \geq 0.$

(*a*) Let s = personal indifferent selling price.



$u(200 + s) = 0.25\ u(200+25) + 0.5\ u(200 + 50\ ) + 0.25\ u(200 - 50)$

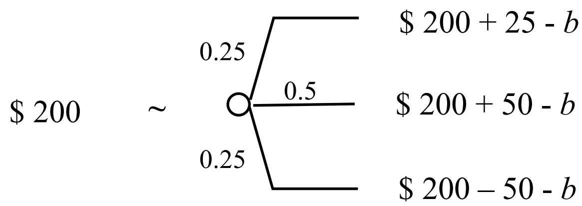
$$\frac{(200+s)^2}{2000} = 0.25\left(\frac{225^2}{2000}\right) + 0.5\left(\frac{250^2}{2000}\right) + 0.25\left(\frac{150^2}{2000}\right)$$

$$(200+s)^2 = 0.25\,(225)^2 + 0.5\,(250)^2 + 0.25\,(150)^2$$

$$s = \$\,22.5562$$

Hence Susan's PISP = **<u>\$22.56</u>**

(*b*) Let b = personal indifferent buying price.



$u(200) = 0.25\ u(200+25 - b) + 0.5\ u(200 + 50 - b) + 0.25\ u(200 - 50 - b)$

$$\frac{200^2}{2000} = 0.25\left(\frac{(225-b)^2}{2000}\right) + 0.5\left(\frac{(250-b)^2}{2000}\right) + 0.25\left(\frac{(150-b)^2}{2000}\right)$$

$$4(200)^2 = (225-b)^2 + 2(250-b)^2 + (150-b)^2$$

$$4b^2 - 1750b + 38125 = 0$$

Solving: b = \$22.99425 (okay) or \$414.5057 (rejected)

Hence Susan's PIBP = **<u>\$22.99</u>**