

TIE4203 Decision Analysis in Industrial & Operations Management

Solutions to Tutorial #9

Question 1 (P9.1)

Computing the main criteria weights w.r.t. Goal:

Goal	Cost	User-friendliness	Software availability	w (exact)	w (RGM)
Cost	1	1/4	1/5	0.09739	0.09739
User-friendliness	4	1	1/2	0.33307	0.33307
Software availability	5	2	1	0.56954	0.56954

$$\lambda_{\max} = 3.0246$$

$$CR = 0.0212 < 0.10$$

Computing the Alternative local weights w.r.t. each main criterion:

Cost	Computer 1	Computer 2	Computer 3	w (exact)	w (RGM)
Computer 1	1	3	5	0.64833	0.64833
Computer 2	1/3	1	2	0.22965	0.22965
Computer 3	1/5	1/2	1	0.12202	0.12202

$$\lambda_{\max} = 3.0037$$

$$CR = 0.00318 < 0.10$$

User-friendliness	Computer 1	Computer 2	Computer 3	w (exact)	w (RGM)
Computer 1	1	1/3	1/2	0.14662	0.14662
Computer 2	3	1	5	0.65707	0.65707
Computer 3	2	1/5	1	0.19631	0.19631

$$\lambda_{\max} = 3.16323$$

$$CR = \underline{0.14072} > 0.10$$

Software availability	Computer 1	Computer 2	Computer 3	w (exact)	w (RGM)
Computer 1	1	1/3	1/7	0.08096	0.08096
Computer 2	3	1	1/5	0.18839	0.18839
Computer 3	7	5	1	0.73064	0.73064

$$\lambda_{\max} = 3.06489$$

$$CR = 0.05594 < 0.10$$

Global Weights:

Alternative	w (exact)	w (RGM)
Computer 1	0.15809	0.15809
Computer 2	0.34851	0.34851
Computer 3	0.49340	0.49340

Note that the RGM approximation method give very accurate results because the matrixes are all of size 3. This will not be so when the matrix sizes are large.

- (a) The company should choose Computer 3 which has the highest global weight.
- (b) Only the pair-wise comparison matrix for “user friendliness” has $CR > 10\%$.
- (c) Transitivity relation is satisfied for all matrices.

Question 2 (P9.2)

(a) Computing the main criteria weights:

	Dependability	Qualification	Experience	Quality	w
Dependability	1	2	3	4	0.46730
Qualification	1/2	1	2	3	0.27718
Experience	1/3	1/2	1	2	0.16009
Quality	1/4	1/3	1/2	1	0.09543

$$\lambda_{\max} = 4.03098, \text{ CR} = 0.011475 < 0.10$$

The weights for the four criteria are:

- Dependability = 0.46730
- Qualification = 0.27718
- Experience = 0.16009
- Quality = 0.09543

Weights for Ratings under Dependability:

	Outstanding	Average	Unsatisfactory	w	Idealized
Outstanding	1	3	7	0.66942	1
Average	1/3	1	3	0.24264	0.36246
Unsatisfactory	1/7	1/3	1	0.08795	0.13138

$$\lambda_{\max} = 3.00702, \text{ CR} = 0.006053 < 0.1$$

Weights for Ratings under Qualification:

	Postgraduate	Graduate	Non-graduate	w	Idealized
Postgraduate	1	3	5	0.63699	1
Graduate	1/3	1	3	0.25828	0.40548
Non-graduate	1/5	1/3	1	0.10473	0.16441

$$\lambda_{\max} = 3.03851, \text{ CR} = 0.033199 < 0.1$$

Weights for Ratings under Experience:

	Exceptional	Average	Little	w	Idealized
Exceptional	1	5	9	0.75140	1
Average	1/5	1	3	0.17818	0.23713
Little	1/9	1/3	1	0.07042	0.09371

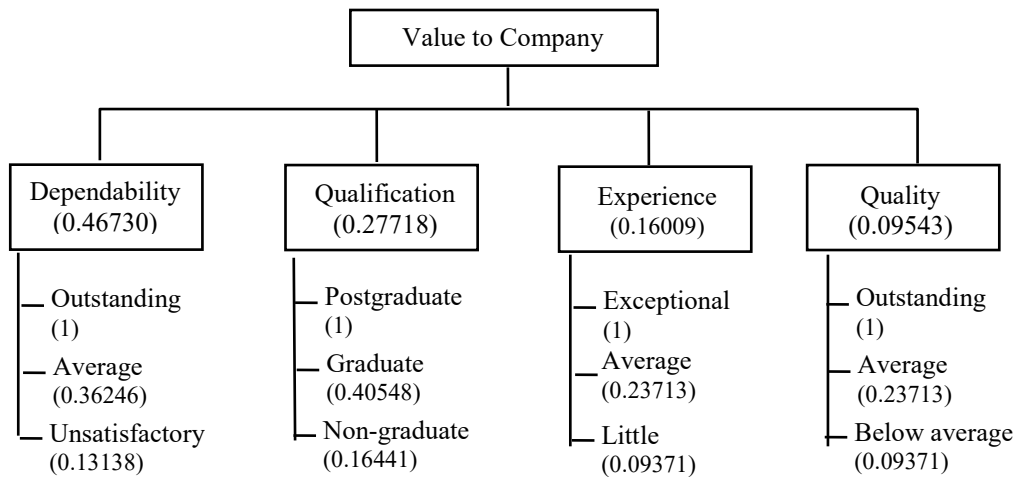
$$\lambda_{\max} = 3.02906, \text{ CR} = 0.025055 < 0.1$$

Weights for Ratings under Quality:

	Outstanding	Average	Below average	w	Idealized
Outstanding	1	5	9	0.75140	1
Average	1/5	1	3	0.17818	0.23713
Below average	1/9	1/3	1	0.07042	0.09371

$$\lambda_{\max} = 3.02906, \text{ CR} = 0.025055 < 0.1$$

The Rating System:



(b) Applying the rating systems to John and Bill:

Candidate	Ratings under Criterion				Overall Rating
	Dependability (0.46730)	Qualification (0.27718)	Experience (0.16009)	Quality (0.09543)	
John	Average (0.36246)	Graduate (0.40548)	Average (0.23713)	Outstanding (1)	0.4145
Bill	Outstanding (1)	Non-graduate (0.16441)	Exceptional (1)	Average (0.23713)	0.6956

- John's overall rating

$$= 0.46730 (0.36246) + 0.27718 (0.40548) + 0.16009 (0.23713) + 0.09543 (1)$$

$$= \mathbf{0.41516}$$
- Bill's overall rating

$$= 0.46730 (1) + 0.27718 (0.16441) + 0.16009 (1) + 0.09543 (0.23713)$$

$$= \mathbf{0.69559}$$
- Hence Bill should get a higher pay increase than John.

Question 3 (P9.3)

(a) With only two alternatives (Investments 1 and 2)

Goal	Expected Return	Degree of Risk	Weight
Expected Return	1	1	0.5
Degree of Risk	1	1	0.5

$$\lambda_{\max} = 2, \text{ CR}=0 < 0.1$$

Expected Return	Investment 1	Investment 2	Weight
Investment 1	1	1/2	0.3333
Investment 2	2	1	0.6667

$$\lambda_{\max} = 2, \text{ CR}=0 < 0.1$$

Degree of Risk	Investment 1	Investment 2	Weight
Investment 1	1	3	0.7500
Investment 2	1/3	1	0.2500

$$\lambda_{\max} = 2, \text{ CR}=0 < 0.1$$

Alternative	Global Weight
Investment 1	0.54167
Investment 2	0.45833

Conclusion: Investment 1 is preferred to Investment 2.

(b) With 3 alternatives (Investments 1, 2, & 3)

Expected Return	Investment 1	Investment 2	Investment 3	Weight
Investment 1	1	1/2	4	0.307692
Investment 2	2	1	8	0.615385
Investment 3	1/4	1/8	1	0.076923

$$\lambda_{\max} = 3, \text{ CR}=0 < 0.1$$

Degree of Risk	Investment 1	Investment 2	Investment 3	Weight
Investment 1	1	3	1/2	0.3
Investment 2	1/3	1	1/6	0.1
Investment 3	2	6	1	0.6

$$\lambda_{\max} = 3, \text{ CR}=0 < 0.1$$

Alternative	Global Weight
Investment 1	0.303846
Investment 2	0.357692
Investment 3	0.338462

• Conclusion: Investment 2 is now preferred to Investment 1.

(c) Rank Reversal between Investment 1 and Investment 2 has occurred with the introduction of Investment 3 although the pairwise comparison sub-matrices for the first two alternatives have not changed.

(d) Using the Ideal Mode

Goal	Expected Return	Degree of Risk	Weight
Expected Return	1	1	0.5
Degree of Risk	1	1	0.5

$$\lambda_{\max} = 2, \text{ CR}=0 < 0.1$$

With only two alternatives (Investments 1 and 2)

Expected Return	Investment 1	Investment 2	Distributive	Ideal
Investment 1	1	1/2	0.3333	0.5000
Investment 2	2	1	0.6667	1.0000

Degree of Risk	Investment 1	Investment 2	Distributive	Ideal
Investment 1	1	3	0.7500	1.0000
Investment 2	1/3	1	0.2500	0.3333

- Global weights under Ideal Mode before normalization

Alternative	Global Weight
Investment 1	0.7500
Investment 2	0.6667

- Conclusion: Investment 1 is preferred to Investment 2 under Ideal Mode.

With 3 alternatives (Investments 1, 2, & 3)

Expected Return	Investment 1	Investment 2	Investment 3	Distributive	Ideal
Investment 1	1	1/2	4	0.307692	0.5
Investment 2	2	1	8	0.615385	1
Investment 3	1/4	1/8	1	0.076923	0.125

Degree of Risk	Investment 1	Investment 2	Investment 3	Distributive	Ideal
Investment 1	1	3	1/2	0.3	0.5
Investment 2	1/3	1	1/6	0.1	0.16667
Investment 3	2	6	1	0.6	1

- Global weights under Ideal Mode before normalization

Alternative	Global Weight
Investment 1	0.5000
Investment 2	0.5833
Investment 3	0.5625

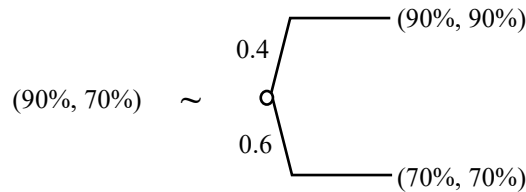
- Investment 2 is now preferred to Investment 1.
- Ideal Mode did not prevent rank reversal for this problem.
- Hence Ideal model does not guarantee rank reversal.

Question 4 (P10.1)

(a) Given $u(x_1, x_2) = k_1 u_1(x_1) + (1 - k_1) u_2(x_2)$

$$70\% \leq x_1 \leq 90\% \text{ and } 70\% \leq x_2 \leq 90\%.$$

$$\begin{aligned} \text{Let } u(70\%, 70\%) &= u_1(70\%) = u_2(70\%) = 0 \\ u(90\%, 90\%) &= u_1(90\%) = u_2(90\%) = 1 \end{aligned}$$

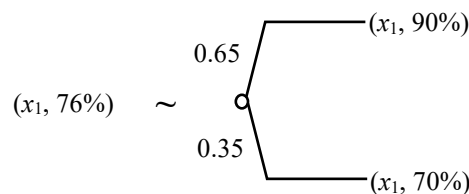


$$\begin{aligned} \Rightarrow 0.4 u(90\%, 90\%) + 0.6 u(70\%, 70\%) &= u(90\%, 70\%) \\ 0.4 u(90\%, 90\%) + 0.6 u(70\%, 70\%) &= k_1 u_1(90\%) + (1 - k_1) u_2(70\%) \\ 0.4 (1) + 0.6 (0) &= k_1 (1) + (1 - k_1) (0) \end{aligned}$$

$$\Rightarrow k_1 = 0.4$$

- Hence the Ministry two-attribute additive utility function is $u(x_1, x_2) = 0.4 u_1(x_1) + 0.6 u_2(x_2)$

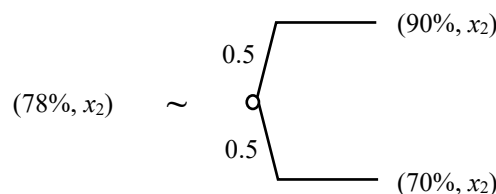
(b)



$$\Rightarrow 0.65 [k_1 u_1(x_1) + k_2 u_2(90\%)] + 0.35 [k_1 u_1(x_1) + k_2 u_2(70\%)] = k_1 u_1(x_1) + k_2 u_2(76\%)$$

$$\underline{k}_1 u_1(x_1) + 0.65 k_2 u_2(90\%) + 0.35 k_2 u_2(70\%) = \underline{k}_1 u_1(x_1) + k_2 u_2(76\%)$$

$$\text{Hence } \underline{u}_2(76\%) = 0.65.$$

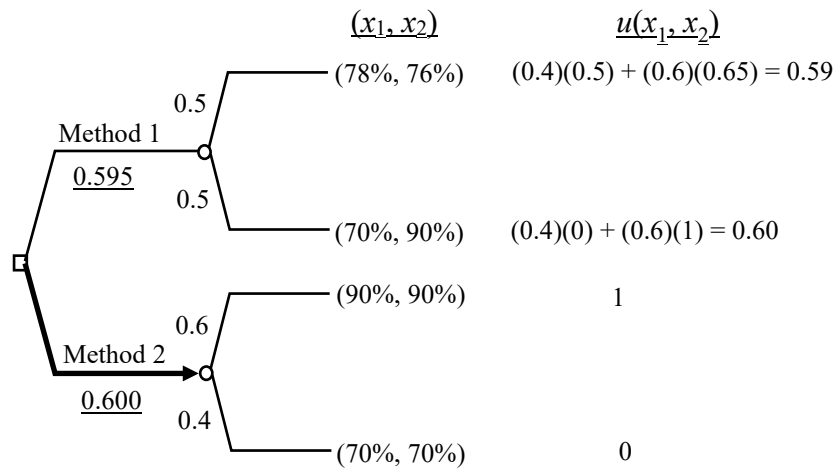


$$\Rightarrow 0.5 [k_1 u_1(90\%) + k_2 u_2(x_2)] + 0.5 [k_1 u_1(70\%) + k_2 u_2(x_2)] = k_1 u_1(78\%) + k_2 u_2(x_2)$$

$$0.5 k_1 u_1(90\%) + k_2 u_2(x_2) + 0.5 k_1 u_1(70\%) = k_1 u_1(78\%) + k_2 u_2(x_2)$$

$$\text{Hence } u_1(78\%) = 0.5.$$

- The decision tree for the two teaching techniques is as follows:



Conclusion: Method 2 is preferred to Method 1.

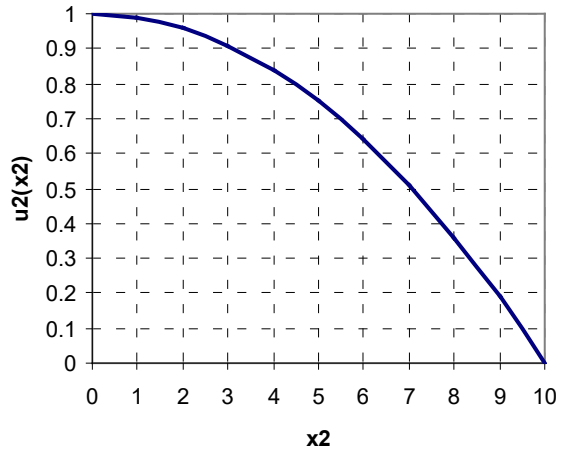
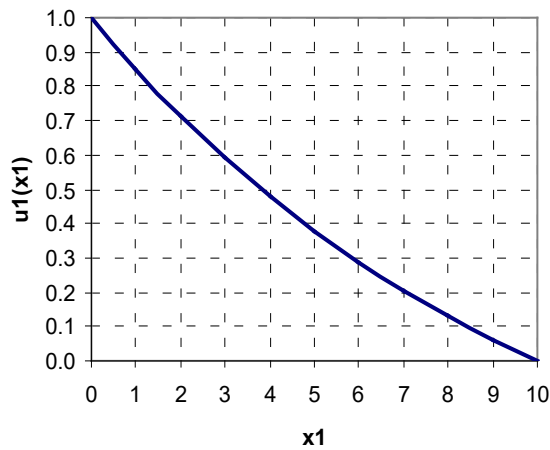
Question 5 (P10.2)

x_1 = Weekly blood shortage ($0 \leq x_1 \leq 10$)

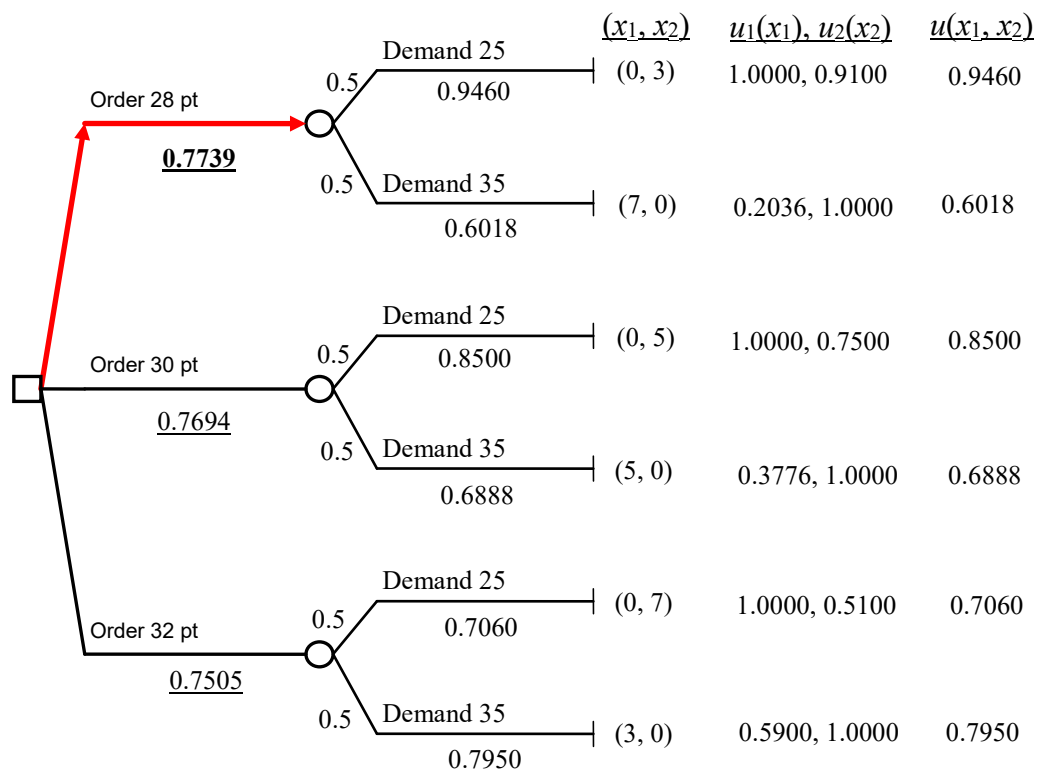
x_2 = Weekly blood outdated ($0 \leq x_2 \leq 10$)

$$u(x_1, x_2) = 0.4 u_1(x_1) + 0.5 u_2(x_2) + 0.1 u_1(x_1) u_2(x_2)$$

where $u_1(x_1) = 0.582 \left[\exp\left(1 - \frac{x_1}{10}\right) - 1 \right]$ and $u_2(x_2) = 1 - \frac{x_2^2}{100}$.



The decision tree is



Conclusion: The hospital should order 28 pints of blood weekly.

Question 6 (P10.4)

(a) The weights for the main criteria with respect to Goal are computed:

	Human prod	Economics	Design	Operations	Exact w	RGM
Human Productivity	1	3	3	7	0.513052	0.5159
Economics	1/3	1	2	5	0.246592	0.2474
Design	1/3	1/2	1	7	0.193575	0.1903
Operations	1/7	1/5	1/7	1	0.046781	0.0463

$$\lambda_{\max} = 4.212088, \text{ CR} = 0.078551 < 10\%$$

The local weights for the alternatives with respect to each criterion are computed:

Human Productivity:

	System A	System B	System C	Exact w	RGM
System A	1	3	5	0.648329	0.6483
System B	1/3	1	2	0.229651	0.2297
System C	1/5	1/2	1	0.122020	0.1220

$$\lambda_{\max} = 3.003695, \text{ CR} = 0.003185 < 10\%$$

Economics:

	System A	System B	System C		RGM
System A	1	1/3	1/2	0.157056	0.1571
System B	3	1	3	0.593634	0.5936
System C	2	1/3	1	0.249311	0.2493

$$\lambda_{\max} = 3.053622, \text{ CR} = 0.046225 < 10\%$$

Design:

	System A	System B	System C	Exact w	RGM
System A	1	1/2	1/7	0.093813	0.0938
System B	2	1	1/5	0.166593	0.1666
System C	7	5	1	0.739594	0.7396

$$\lambda_{\max} = 3.014152, \text{ CR} = 0.012200 < 10\%$$

Operations:

	System A	System B	System C	Exact w	RGM
System A	1	3	1/5	0.178178	0.1782
System B	1/3	1	1/9	0.070418	0.0704
System C	5	9	1	0.751405	0.7514

$$\lambda_{\max} = 3.029064, \text{ CR} = 0.025055 < 10\%$$

The composite weights for the three alternative systems are:

- System A: $(0.513052)(0.648329) + (0.246592)(0.157056) + (0.093813)(0.0938) + (0.046781)(0.178178)$
 $= 0.397850$
- System B: $(0.513052)(0.229651) + (0.246592)(0.593634) + (0.193575)(0.166593) + (0.046781)(0.070418)$
 $= 0.299750$
- System C: $(0.513052)(0.122020) + (0.246592)(0.249311) + (0.193575)(0.739594) + (0.046781)(0.751405)$
 $= 0.302399$

Hence System A should be chosen as it has the highest global weight.

(b)

Alternative	Effectiveness	EUAC (\$)
System B	0.397850	80,000
System A	0.299750	100,000
System C	0.302399	110,000

- Efficient Cost-Effective Alternatives are B and A.
- The efficient frontier is shown below:

